DYNAMIC DEBT SUSTAINABILITY ANALYSIS – AFRICA

Stochastic Debt Forecasting

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Abstract

This paper develops a stochastic debt sustainability framework for African economies that integrates macroeconomic uncertainty, endogenous fiscal policy, and dynamic feedback effects within a unified forecasting system. The framework jointly estimates a fiscal reaction function and macroeconomic equations using Iterative Seemingly Unrelated Regressions and generates probabilistic debt trajectories through Monte Carlo simulations incorporating stochastic shocks and parameter uncertainty. Applied to Kenya, the analysis examines how fiscal behavior, external shocks, and the composition of public expenditure shape medium-term debt dynamics. The results show that fiscal policy responds to rising debt in a stabilizing direction but remains procyclical and vulnerable to external shocks. Importantly, the findings identify a productive-transformation channel through which higher development expenditure supports economic growth and can improve debt sustainability despite short-term deterioration in the primary balance. Protecting productive expenditure during fiscal consolidation also strengthens the fiscal response to debt. Extending the analysis to the broader African context, the results confirm that productive transformation investments are associated with stronger economic growth and lower public debt, while external conditions, particularly global demand, remain an important source of debt and macroeconomic volatility across the continent. The paper demonstrates that debt sustainability depends not only on the size of fiscal adjustment, but also on expenditure composition, macroeconomic feedbacks, and external conditions, providing policymakers with a flexible framework for evaluating fiscal risks and policy trade-offs.

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INTRODUCTION

Conventional debt sustainability analysis (DSA) presents a limited view of debt dynamics. Conventional DSAs typically rely on deterministic baseline scenarios that treat fiscal policy and macroeconomic conditions as exogenous or fixed. While operationally straightforward, these methods fail to adequately capture the dynamic and uncertain nature of debt evolution, particularly in environments characterized by macroeconomic volatility and active policy responses.

This paper adopts a stochastic debt sustainability framework designed to address these limitations. Specifically, the approach explicitly incorporates macroeconomic uncertainty, allows for the endogeneity of fiscal policy, captures feedback effects between fiscal policy and key macroeconomic variables, and generates probability distributions of debt outcomes rather than single-point estimates. In doing so, it aligns with and extends methodologies employed by institutions such as the International Monetary Fund (IMF), the World Bank and central banks, enhancing the use of fan charts for debt sustainability and macroeconomic risk analysis (Romeu & Kawakami, 2011) (Debreu and Romeu, 2026).

The objective of the model is to assess risks surrounding projected debt trajectories over a five-year horizon through a unified estimation strategy that jointly models fiscal and macroeconomic dynamics. Unlike standard frameworks, this approach explicitly accounts for the bidirectional relationship between fiscal policy and macroeconomic outcomes. Fiscal policy responds contemporaneously to macroeconomic shocks through automatic stabilizers and discretionary policy actions, while macroeconomic variables evolve in part as a function of past fiscal positions.

Our stochastic dynamic debt sustainability analysis framework embraces both macroeconomic interdependence and uncertainty. The framework incorporates policy transmission mechanisms by introducing lags in the effects of fiscal policy on macroeconomic aggregates, reflecting delays in policy recognition, implementation, and transmission, as well as legislative uncertainty. Structural fiscal policy shocks are identified under the assumption that changes in the structural fiscal balance affect macroeconomic conditions with a lag, enabling a more realistic representation of policy impact over time. Uncertainty is modelled explicitly through the stochastic properties of the estimated parameters, allowing for non-normal shock distributions and potential structural changes in the underlying data-generating process. As a result, debt projections are derived from jointly simulated paths of macroeconomic variables and primary fiscal balances, capturing both contemporaneous correlations and dynamic feedback effects.

The remainder of this paper outlines the methodological framework, estimation strategy, and simulation approach used to generate probabilistic debt forecasts. It also provides guidance on the implementation of the model, including the use of Monte Carlo simulations to evaluate debt dynamics under alternative macroeconomic scenarios.

OBJECTIVES OF THE FRAMEWORK

The framework is designed to generate probabilistic projections of public debt dynamics by jointly modelling fiscal behaviour and macroeconomic conditions, including the role of external shocks. By moving beyond deterministic forecasting approaches, it provides a more comprehensive and policy-relevant assessment of debt sustainability under uncertainty.

A central objective of the framework is to quantify the full distribution of possible debt trajectories over the projection horizon. Rather than relying on single-point estimates, the model produces a range of outcomes that reflect the stochastic nature of macroeconomic shocks, parameter uncertainty, and policy responses. The framework also aims to identify and quantify the key macroeconomic drivers of debt sustainability risks. Explicitly modelling the interaction between fiscal policy and macroeconomic variables (such as growth, inflation, interest rates, exchange rates, and development expenditures) allows for a decomposition of debt dynamics into their underlying determinants. This facilitates a clearer understanding of the channels through which shocks propagate and affect fiscal outcomes.

Another core objective is to evaluate the sensitivity of debt dynamics to macroeconomic shocks in a way that the interrelationships among key macroeconomic drivers are considered. Through simulation-based analysis, the framework assesses how different types of shocks, both domestic and external, affect debt trajectories under alternative scenarios. This includes capturing nonlinearities and feedback effects that are typically absent in standard deterministic frameworks.

To achieve these objectives, the framework incorporates several key features that allow for a better treatment of uncertainty. Debt projections explicitly account for stochastic macroeconomic shocks, allowing for realistic fluctuations in economic conditions. In addition, uncertainty in estimated parameters is embedded in the simulation process, ensuring that projections reflect both model and data limitations. Crucially, the framework models fiscal policy as endogenous, capturing how fiscal authorities respond to evolving macroeconomic conditions through both automatic stabilizers and discretionary measures.

Taken together, these elements allow the framework to produce internally consistent and empirically grounded projections of debt dynamics for both unconditional and conditional forecasting. The resulting outputs support a more nuanced assessment of fiscal risks, enhance the analytical toolkit for debt sustainability analysis, and provide a robust basis for policy evaluation under uncertainty. By simulating a large number of potential macro-fiscal paths, the model produces fan charts and risk metrics that quantify the probability of debt breaching critical thresholds. This enhances the ability to assess fiscal vulnerabilities and to design policies that are robust to adverse scenarios.

APPLIED ANALYSIS: KENYA

Applying this forecast and analysis to Kenya serves as a prototype for other African economies facing similar macro-fiscal conditions, including exposure to external shocks, evolving fiscal frameworks, and constrained fiscal space. By combining country-specific data with a unified modelling approach, the Kenya case demonstrates how the framework can be adapted across contexts to produce risk-informed debt projections. Its flexibility allows for calibration to different institutional settings and data environments, enabling policymakers to assess expected debt paths and the likelihood of adverse outcomes. The application focuses on evaluating medium-term debt dynamics and quantifying fiscal risks under uncertainty.

The country-specific calibration of the model is based on Kenyan fiscal and macroeconomic data as well as external variables. Domestic data includes historical series on debt (both domestic and external) primary balances, and productive transformation spending; as well as a set of macroeconomic indicators such as GDP growth, exchange rate depreciations, inflation, and real interest rates. External variables that influence debt dynamics (such as global interest rates and oil prices) are also incorporated to capture Kenya's exposure to international economic shocks.

Applying the framework to Kenya illustrates how the framework can be used to generate internally consistent, data-driven, and policy-relevant assessments of debt sustainability analysis for African economies. By integrating stochastic simulations with endogenous fiscal behaviour, the model provides a robust platform for evaluating fiscal risks and informing medium-term fiscal strategy in a context of uncertainty.

METHODOLOGY

The stochastic debt forecasting framework is based on the joint estimation of a fiscal reaction function and a macroeconomic system using Iterative Seemingly Unrelated Regressions (SUR). This approach allows for the simultaneous estimation of fiscal and macroeconomic equations while accounting for cross-equation error correlations. A defining feature of the framework is the explicit modelling of feedback effects from fiscal policy to macroeconomic variables, ensuring that both evolve within a unified system. At the same time, the framework allows for external variables to affect the domestic economy but does not allow for any feedback, consistent with being a small open economy. The empirical estimation relies on annual data over a sufficiently long historical sample to capture stable macroeconomic relationships.

Forecasts are generated through a Monte Carlo simulation procedure designed to produce a distribution of possible future paths that are consistent with the main model structure. The simulation proceeds by drawing parameters from the estimated coefficient distributions, simulating macroeconomic variables using the autoregressive multi-equation system, generating fiscal outcomes through the fiscal reaction function, and applying the debt motion equation to obtain the corresponding debt trajectories. Repeating this process thousands of times yields a probability distribution of future debt and macroeconomic outcomes, with parameter uncertainty explicitly incorporated at each stage.

The identification of structural fiscal shocks in this framework follows a logic analogous to that used in structural vector autoregressions (SVARs) in monetary policy analysis, where identifying restrictions are required to isolate exogenous factors from endogenous responses of fiscal variables and macroeconomic conditions (Mertens & Ravn, 2011). Specifically, restrictions are imposed on the fiscal reaction function to decompose the primary balance into structural and cyclical components, consistent with standard fiscal approaches that separate underlying policy from cyclical fluctuations (EPRS, 2023; IMF, 2025). These components play a central role in the simulation process, driving the joint evolution of macroeconomic aggregates and debt dynamics. The resulting distribution of debt outcomes can be summarized using fan charts, providing a transparent tool for fiscal risk analysis.

A key contribution of the framework lies in its treatment of uncertainty. Forecast uncertainty arises not only from stochastic shocks but also from uncertainty in estimated parameters. Accordingly, projections are generated using repeated simulations based on random draws from the full distribution of estimated coefficients rather than relying on point estimates. Additional restrictions on the macroeconomic system help reduce noise in the projections, while the framework avoids imposing normality assumptions on shocks and explicitly accounts for the possibility of structural breaks (Romeu & Debrun, 2026).

Finally, by endogenizing fiscal policy within the macroeconomic system, the framework captures nonlinear feedback effects that can significantly influence both the central tendency and dispersion of debt projections. These dynamics are economically meaningful: they allow for the identification of both virtuous cycles where credible fiscal adjustments improve macroeconomic outcomes, and adverse scenarios in which weak fiscal responses amplify vulnerabilities. Ignoring such interactions would risk understating both the potential gains from sound macroeconomic policy and the risks associated with fiscal slippage.

A note on identification is warranted given the central role of structural fiscal shocks in the simulation framework. The approach taken here follows the recursive identification logic established in Blanchard and Perotti (2002), which exploits the institutional observation that discretionary fiscal policy cannot respond to macroeconomic shocks within the same period due to decision, legislative, and implementation lags. Under this assumption, changes in the structural primary balance affect macroeconomic aggregates with a lag, while macroeconomic variables can influence fiscal outcomes contemporaneously through automatic stabilizers. This recursive structure provides the identifying restriction that separates endogenous fiscal responses from exogenous policy innovations in the joint system. The assumption is particularly defensible in the Kenyan institutional context, where budget execution lags are well-documented and mid-year discretionary fiscal adjustments are constrained by legislative procedures and IMF program conditionality, reducing the likelihood of same-period fiscal responses to macroeconomic shocks. The Mertens and Ravn (2011) framework, cited in the methodology, uses a complementary but distinct identification strategy based on narrative external instruments derived from legislative records, an approach that is not directly applicable here given data availability constraints in the Kenyan context, but whose conceptual motivation for isolating exogenous fiscal innovations is consistent with the recursive approach adopted. The sensitivity of the results to this identifying assumption is partially addressed through the bootstrap simulation procedure, which propagates parameter uncertainty through the full system rather than conditioning on point estimates. A fully robust assessment would additionally test alternative orderings, for instance, allowing macroeconomic variables to respond contemporaneously to fiscal shocks, and comparing the resulting impulse response functions. This extension is left for future work, and users of the framework in alternative country contexts should assess whether the lag assumption remains institutionally appropriate given country-specific budget execution characteristics.

STRUCTURE OF THE MODEL

The stochastic debt sustainability analysis framework consists of three interdependent components. These components are jointly estimated to capture the dynamic, two-way interaction between fiscal policy and macroeconomic outcomes: (I) a fiscal reaction function, (ii) a macroeconomic dynamics block, and (iii) a debt motion equation. Together, these blocks form an integrated system in which fiscal and macroeconomic variables evolve endogenously and consistently over time.

Fiscal Reaction Function

The first component, the fiscal reaction function, models the primary balance as a function of lagged values as well as debt-to-GDP and key macroeconomic fundamentals. This specification captures both cyclical responses, through automatic stabilizers, and structural fiscal behaviour. The function is estimated using a long historical sample (1981-2025).

The Fiscal Reaction Function presents the endogenous response of the primary balance to internal and external macroeconomic conditions. The function models the primary balance (denoted *pb*) as a function of the lagged primary balance and macroeconomic variables, capturing the systematic response of fiscal policy to economic conditions and debt dynamics. Specifically, the primary balance is estimated as a function of the lagged debt-to-GDP ratio (*debt*), the nominal exchange rate depreciation (xr, defined such that higher values indicate a depreciation of the domestic currency against the US dollar), the output gap (*gap*, measured as the deviation from HP-filtered trend output), and commodity price movements proxied by oil price growth (*oil*). A structural dummy variable (*dum*) is included to control for periods characterized by unusually high primary deficits. This dummy accounts for structural breaks associated with measurement changes and major economic disruptions and does not reflect normative judgments about fiscal performance.

To examine the role of productive transformation expenditure in public debt sustainability in African economies, the model evaluates whether higher allocations to development spending improve debt outcomes. Specifically, the share of development expenditures as a percentage of GDP (exp_dev) is incorporated into the framework, and alternative scenarios, low, constant, and high, are simulated to capture different expenditure paths over time. These scenarios allow for a systematic assessment of how changes in the composition of public spending affect macroeconomic performance and debt dynamics. This approach is used to test the following hypothesis:

- **H1 (Productive Transformation Expenditure):** Debt is more sustainable when governments allocate a higher share of resources to productive transformation.

To further examine the role of expenditure composition in debt sustainability, the model tests whether debt sustainability depends on how governments allocate productive transformation spending during different phases of the fiscal cycle. To do this, the model incorporates two policy indicators (Protect and Prioritize), where the first indicates whether protecting productive transformation expenditure during fiscal consolidations affects debt sustainability while the second indicates whether increasing productive transformation expenditure during fiscal expansions affects debt sustainability. These variables are used to evaluate two hypotheses:

- **H2 (Consolidation):** Debt is more sustainable when governments protect productive sectors during fiscal adjustment. This is captured by the variable (*prot*).
- **H3 (Expansion):** Debt is more sustainable when governments prioritize productive sectors during fiscal expansion. This is captured by the variable (*prior*).

A positive relationship between previous debt and primary balance reflects a debt stabilizing mechanism. Higher primary balances when previous debt is higher indicate the effort that fiscal authorities make to prevent debt from being unsustainable, by generating the resources for debt repayment. In the case of hypothesis 2 and 3, the interaction terms between these indicators and lagged debt are included to assess whether the strength of the primary balance response to previous debt varies across expenditure-composition regimes.

For Kenya, the fiscal reaction function controlling for these changes in measures is:

$$\boldsymbol{pb_t} = \lambda_0 + \lambda_1(gap_{t-1} \cdot dum_t) + \lambda_2 pb_{t-1} + \lambda_3(xr_{t-1} \cdot dum_t) + \lambda_4 debt_{t-1} + \lambda_5 exp\_dev_{t-1} + \lambda_6 oil_{t-1} + \lambda_7(debt_{t-1} \cdot prot_t) + \lambda_8(debt_{t-1} \cdot prior_t)$$

λ_7>0 supports H2

λ_8>0 supports H3

Where:

prot = 1 if $Consolidation_t = 1$ and $exp\_dev\_sh_t \geq exp\_dev\_sh_{t-1}$

prior = 1 if $Expansion_t = 1$ and $exp\_dev\_sh_t > exp\_dev\_sh_{t-1}$

exp_dev_sh = the development expenditure share of total expenditure.

Macroeconomic Dynamics

The second component, the macroeconomic dynamics block, models the joint evolution of economic fundamentals. Together with the first component, they are estimated using a restricted Autoregressive Multi-equation SUR model, ensuring internal consistency and allowing for correlated disturbances across equations.

The system includes both domestic and external variables that are central to debt dynamics. External variables include the U.S. real interest rate (*ius_r*) and oil price growth (*oil*), which capture global financial conditions and commodity price shocks. Domestic variables include Kenya's real interest rate (*iken_r*), real GDP growth (*gdp*), nominal exchange rate depreciation (*xr*), and development expenditures as a percentage of GDP (*exp_dev*). Variables treated as exogenous are US interest rates, oil prices, and government development expenditures, all other variables are treated as endogenous in the model. Two dummy variables are incorporated to capture structural disruptions. The structural dummy (*dum*) controls for periods associated with unusually high primary deficits and potential structural breaks, while a COVID-19 dummy (*covid*) captures the global economic disruption associated with the pandemic. The macroeconomic system is specified as follows:

$$\boldsymbol{ius\_r_t} = \alpha_0 + \alpha_1 ius\_r_{t-1}$$

$$\boldsymbol{oil_t} = \beta_0 + \beta_1 oil_{t-1} + \beta_2 (oil_{t-1} \cdot covid)$$

$$\boldsymbol{iken\_r_t} = \gamma_0 + \gamma_1 iken\_r_{t-1} + \gamma_2 xr_{t-1}$$

$$\boldsymbol{gdp_t} = \delta_0 + \delta_1 gdp_{t-1} + \delta_2 (gdp_{t-1} \cdot dum_t) + \delta_3 xr_{t-1} + \delta_4 exp\_dev_{t-1}$$

$$\boldsymbol{xr_t} = \theta_0 + \theta_1 gdp_{t-1} + \theta_2 xr_{t-1} + \theta_3 xr_{t-1}^2$$

Debt Motion Equation

The third component, the debt motion equation, links fiscal and macroeconomic outcomes to the evolution of the debt-to-GDP ratio. This accounting relationship incorporates primary balances, interest-growth differentials, and exchange rate effects to determine debt dynamics over time. By taking as inputs the simulated fiscal balances and macroeconomic variables, the debt equation produces corresponding debt trajectories. Crucially, it propagates uncertainty from both shocks and parameter estimates, ensuring that debt projections fully reflect the stochastic nature of the system.

Debt dynamics in the framework follow the standard accounting identity linking public debt, economic growth, interest rates, and the primary balance. Consistent with the literature, the evolution of the debt-to-GDP ratio is governed by the following motion equation:

$$\boldsymbol{debt_t} = \left(\frac{(1 + ius\_r_t)(1 + xr_t) \cdot dus_t \cdot debt_{t-1} + (1 + iken\_r_t)(1 - dus_t) \cdot debt_{t-1}}{1 + gdp_t} \right) - pb_t$$

where *dus* denotes the share of external (foreign currency or dollar-linked) debt in total public debt.

This formulation decomposes debt dynamics into external and domestic components, allowing for differential borrowing costs and exchange rate effects. External debt is affected by both global real interest rates and exchange rate movements, while domestic debt evolves according to the domestic real interest rate. The denominator captures the effect of economic growth in diluting the debt burden, and the primary balance reflects fiscal policy adjustments.

The impact of macroeconomic shocks on debt operates through both direct and indirect channels. Direct effects arise from changes in interest rates, exchange rates, and growth, while indirect effects are transmitted through their influence on fiscal outcomes via the fiscal reaction function. Lagged fiscal variables in particular, such as the primary balance, enter the macroeconomic system, affecting output growth and domestic interest rates, which in turn feed back into debt dynamics. By integrating these channels, the debt motion equation ensures that projected debt paths fully reflect the interaction between macroeconomic shocks, fiscal responses, and structural features of the debt portfolio.

The interaction across these three components is central to the framework. Simulated macroeconomic paths feed into the fiscal reaction function to generate corresponding fiscal balances, which in turn influence future macroeconomic conditions and debt accumulation. This iterative process produces a large number of internally consistent macro-fiscal scenarios, each representing a possible realization of the economy.

The empirical implementation of the model relies on a comprehensive set of fiscal, macroeconomic, and external variables constructed from multiple data sources. Fiscal variables include the primary balance, public debt as a share of GDP, development expenditures (measured both as a share of GDP and total expenditures) the share of external debt, and a structural dummy capturing periods of elevated fiscal deficits. These variables are compiled from different sources including the IMF's World Economic Outlook (WEO), the IMF Historical Public Debt Database, the Central Bank of Kenya, national budget documents, and supplementary studies. Macroeconomic variables comprise real GDP growth, the real interest rate, exchange rate dynamics, and the output gap, primarily sourced from the WEO, World Bank, and Central Bank of Kenya, with additional calculations applied where needed. External variables include global interest rates proxied by U.S. real interest rates (World Bank and FRED), oil price growth (WEO), and a COVID-19 dummy capturing global disruptions. Together, these variables provide a consistent and comprehensive dataset spanning 1981–2025, enabling the joint estimation and simulation of fiscal and macroeconomic dynamics.

ESTIMATION RESULTS

The SUR estimation results highlight several key mechanisms driving debt dynamics, including the role of public investment on productive transformation, in the Kenyan context. The evidence supports the presence of a productive transformation channel, whereby higher development expenditures are associated with increased real GDP growth ($\delta_4 = 0.867$). This finding suggests that public investment plays a critical role in enhancing economic performance, which in turn can improve debt sustainability through higher growth.

The results also confirm the importance of endogenous fiscal policy responses, with some of them serving as debt stabilizers. Higher levels of previous public debt are associated with improvements in the primary balance ($\lambda_4 = 0.029$), indicating that fiscal authorities tend to have less expansionary fiscal policy when experiencing rising debt burdens. At the same time, fiscal policy appears to be counter cyclical ($\lambda_1 = -0.151$), with fiscal balances moving in contrary to the business cycle (higher growth is associated to higher primary deficits). External factors also play a significant role: currency depreciations are found to weaken the primary balance ($\lambda_3 = -0.054$), likely through increased debt servicing costs and inflationary pressures, while increases in oil prices similarly deteriorate fiscal outcomes ($\lambda_6 = -0.007$), reflecting higher import costs and broader macroeconomic stress.

The model also uncovers nonlinear exchange rate dynamics, where previous depreciations tend to lead to future depreciations but at a decreasing rate. This is characterized by a concave relationship between past and current depreciations ($\theta_2 = 0.588$ and $\theta_3 = -0.900$). This implies that the marginal impact of exchange rate movements is not constant. Large depreciations may have disproportionately different effects compared to smaller ones, reflecting potential threshold effects or market adjustments.

Taken together, these results underscore the complex interaction between fiscal policy, macroeconomic conditions, and external shocks. They highlight the dual role of fiscal policy as both a response mechanism to debt accumulation and a transmission channel through which macroeconomic shocks influence debt sustainability. The following table shows the results of the joint estimation of the fiscal policy reaction function and the estimation by iterative seemingly unrelated regression. Notice that the evidence is found to support the existence of this feedback channel.

System: SYSOIL					
Estimation Method: Iterative Seemingly Unrelated Regression					
Date: 03/09/26 Time: 10:50					
Sample: 1982 2025					
Included observations: 44					
Total system (balanced) observations 264					
Simultaneous weighting matrix & coefficient iteration					
Convergence achieved after: 8 weight matrices, 9 total coef iterations					
		Coefficient	Std. Error	t-Statistic	Prob.
C(10)	α_0	0.006072	0.003306	1.836692	0.0675
C(11)	α_1	0.816616	0.068487	11.92366	0
C(100)	β_0	0.02744	0.04003	0.685477	0.4937
C(101)	β_1	0.127835	0.148808	0.859058	0.3912
C(102)	β_2	-1.560794	0.733807	-2.126982	0.0344
C(20)	γ_0	0.037554	0.014607	2.571003	0.0107
C(21)	γ_1	0.35386	0.130955	2.702152	0.0074
C(26)	γ_2	0.161747	0.06401	2.526896	0.0122
C(30)	δ_0	0.015455	0.00833	1.855304	0.0648
C(32)	δ_1	0.039785	0.143878	0.276519	0.7824
C(33)	δ_2	-0.612083	0.174471	-3.508228	0.0005
C(35)	δ_3	-0.038768	0.022237	-1.743388	0.0825
C(37)	δ_4	0.866867	0.213886	4.052945	0.0001
C(40)	θ_0	0.096483	0.043941	2.195752	0.0291
C(41)	θ_1	-1.276439	0.87118	-1.465185	0.1442
C(42)	θ_2	0.588121	0.255166	2.304854	0.022
C(43)	θ_3	-0.900479	0.345587	-2.605653	0.0097
C(50)	λ_0	-0.001005	0.005332	-0.188407	0.8507
C(51)	λ_1	-0.151221	0.05842	-2.588513	0.0102
C(52)	λ_2	0.571974	0.066191	8.641306	0
C(53)	λ_3	-0.053725	0.025626	-2.096471	0.0371
C(54)	λ_4	0.028873	0.007047	4.09729	0.0001
C(55)	λ_5	-0.438341	0.087913	-4.986088	0
C(56)	λ_6	-0.007113	0.003569	-1.993058	0.0474
Determinant residual covariance			3.11E-18		

Equation: IUS_R = C(10) + C(11) * IUS_R(-1)			
Observations: 44			
R-squared	0.754784	Mean dependent var	0.040074
Adjusted R-squared	0.748945	S.D. dependent var	0.02266
S.E. of regression	0.011354	Sum squared resid	0.005414
Durbin-Watson stat	1.808634		

Equation: OIL = C(100) + C(101) *OIL(-1) +C(102)*OIL(-1)*COVID			
Observations: 44			
R-squared	0.07765	Mean dependent var	0.049238
Adjusted R-squared	0.032658	S.D. dependent var	0.267674
S.E. of regression	0.263267	Sum squared resid	2.841697
Durbin-Watson stat	2.069048		

Equation: IKEN_R = C(20) + C(21) * IKEN_R(-1) + C(26) * XR(-1)			
Observations: 44			
R-squared	0.199766	Mean dependent var	0.075855
Adjusted R-squared	0.16073	S.D. dependent var	0.064871
S.E. of regression	0.059429	Sum squared resid	0.144805
Durbin-Watson stat	2.148091		

Equation: GDP = C(30) + C(32) * GDP(-1) + C(33) * GDP(-1)*DUM + C(35) * XR(-1) + C(37)*EXP_DEV(-1)			
Observations: 44			
R-squared	0.237503	Mean dependent var	0.038432
Adjusted R-squared	0.159298	S.D. dependent var	0.022455
S.E. of regression	0.020588	Sum squared resid	0.016532
Durbin-Watson stat	2.167761		

Equation: XR = C(40) + C(41) * GDP(-1) + C(42) * XR(-1) + C(43)*XR(-1)^2			
Observations: 44			
R-squared	0.177941	Mean dependent var	0.069663
Adjusted R-squared	0.116287	S.D. dependent var	0.139493
S.E. of regression	0.131132	Sum squared resid	0.68782
Durbin-Watson stat	2.187771		

Equation: PB = C(50) + C(51)*GAP(-1) *DUM + C(52) * PB(-1)+ C(53) * XR(-1) *DUM + C(54) * DEBT(-1) + C(55)*EXP_DEV_SH(-1) + C(56) * OIL(-1)			
Observations: 44			
R-squared	0.920558	Mean dependent var	-0.00709
Adjusted R-squared	0.907675	S.D. dependent var	0.022505
S.E. of regression	0.006838	Sum squared resid	0.00173
Durbin-Watson stat	1.390191		

Table 1: Iterative Seemingly Unrelated Regression Estimation Results

UNCONDITIONAL FORECAST RESULTS AND UNCERTAINTY

The model generates a set of baseline (unconditional forecast) results that summarize the unconditional forecast of the joint macro-fiscal system. These results are derived from 10,000 Monte Carlo simulations, which produce a distribution of possible future paths for key variables: debt-to-GDP, primary balance, real GDP growth, domestic real interest rates, the output gap, exchange rate movements, changes in oil prices, and global real interest rates (proxied by US interest rates). The baseline scenario corresponds to the median (50th percentile) of these simulated outcomes.

The results are presented using fan charts, which provide a visual representation of the uncertainty around the projected debt and other macroeconomic indicators paths. These charts display the median projection alongside confidence intervals derived from simulation percentiles, as well as potential asymmetries in the distribution. The dispersion of the fan chart bands reflects the combined effects of macroeconomic volatility, uncertainty in fiscal policy responses, and parameter estimation uncertainty embedded in the model.

The baseline fan chart for the debt-to-GDP ratio indicates a gradual declining trend, with the median falling to approximately 62.47 percent of GDP over the projection horizon. As figure 2 shows, the distribution widens over time (as uncertainty over the results increases over time) and is skewed to the downside, indicating a greater dispersion toward lower debt outcomes. There remains a non-negligible probability of more adverse debt trajectories, driven by unfavourable macroeconomic conditions or weaker fiscal responses.

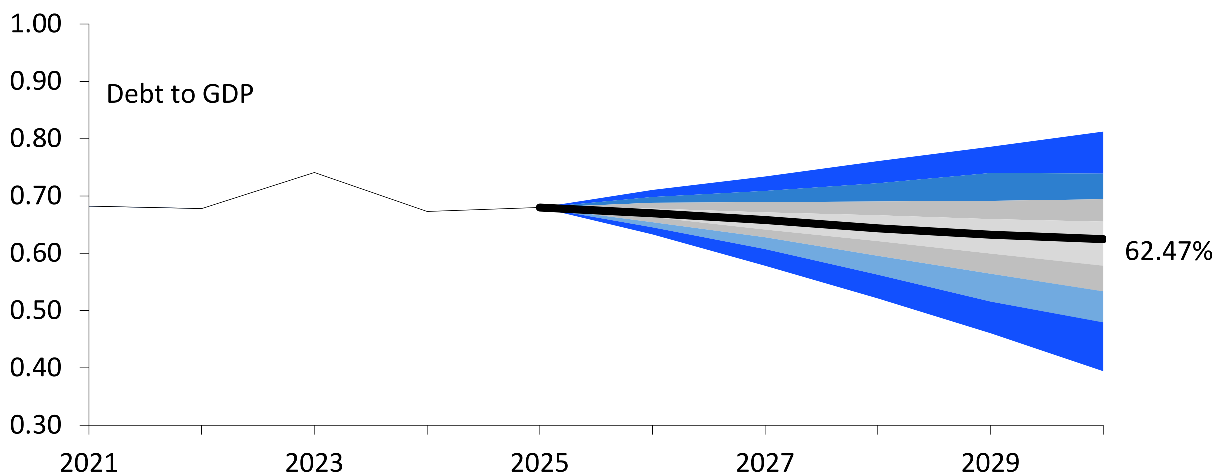

Figure 1: Fan Chart of Debt to GDP (Baseline)

PRODUCTIVE TRANSFORMATION EXPENDITURES AND DEBTS SUSTAINABILITY

Variable	Low	Current	High
debt	63.18	62.47	61.58
xr	-1.50	-4.10	-6.64
gap	-1.09	3.35	7.87

gdp	3.75	5.63	7.45
pb	1.04	-0.41	-1.86
iken_r	6.65	6.44	6.16

Table 2: Productive Expenditure Scenario Results

As mentioned in the methodology, our first hypothesis (H1) tests whether increasing productive transformation expenditure affects the sustainability of public debt in African economies. The results of three productive expenditure scenarios (low, constant, and high) provide strong support for H1, indicating that higher development expenditures contribute to improved debt sustainability through their impact on macroeconomic performance, including higher economic growth.

The three scenarios illustrate how changes in the share of development spending shape both fiscal and economic outcomes. In the low scenario, where development expenditure declines by 0.5 percentage points annually, the economy experiences weaker growth (3.75%) and a negative output gap (-1.09), indicating underutilized productive capacity. Although the primary balance improves (1.04%), reflecting tighter fiscal conditions, the debt-to-GDP ratio increases to 63.18%. This suggests that fiscal consolidation achieved through reduced productive spending may come at the cost of lower growth, ultimately worsening debt dynamics. In the constant scenario, development expenditure remains constant, resulting in moderate growth (5.63%) and a positive output gap (3.35). The primary balance remains slightly negative (-0.41%), and the debt-to-GDP ratio stabilizes at 62.47%. This scenario reflects a continuation of current policies, yielding relatively stable but unremarkable improvements in debt sustainability. In the high scenario, where development expenditure increases by 0.5 percentage points annually, the economy benefits from significantly stronger growth (7.45%) and a substantially higher output gap (7.87), indicating robust economic expansion. While the primary balance deteriorates further (-1.86%) due to increased spending, the debt-to-GDP ratio declines to 61.58%. This improvement is supported by stronger growth, an appreciating exchange rate, and lower domestic interest rates, all of which contribute to more favourable debt dynamics.

As exp_dev ↑ ⇒ debt ↓

⇒ pb ↓ (as expected)

⇒ gdp ↑, gap ↑

⇒ xr ↓

Figure 2: Productive Transformation Effects

The results highlight a key trade-off: increasing development expenditures may weaken short-term fiscal balances but generates growth-enhancing effects that improve debt sustainability over time. The findings suggest that prioritizing productive transformation can create a virtuous cycle in which higher investment boosts economic performance, reduces borrowing costs, and ultimately leads to more sustainable debt outcomes.

FISCAL ALLOCATION, PRODUCTIVE EXPENDITURE AND DEBTS SUSTAINABILITY

This section evaluates whether the composition of public expenditure influences the sustainability of public debt in African economies, specifically testing H2 (consolidation) and H3 (expansion). The results provide mixed evidence on whether the composition of public expenditure affects debt sustainability through fiscal behaviour. The positive coefficient on the interaction term between lagged debt and the Protect indicator (λ_7 = 0.008786) supports H2, suggesting that during periods of fiscal consolidation, maintaining or increasing the share of development expenditures strengthens the fiscal response to rising debt. In practical terms, this implies that protecting productive sectors during adjustment is associated with an improvement in the primary balance for a given level of debt, thereby enhancing debt sustainability. In contrast, the negative coefficient on the interaction term with the Prioritize indicator (λ_8 = −0.00739) does not support H3, indicating that increasing the share of development expenditures during expansionary periods weakens the fiscal response to debt. This may reflect that, during expansions, prioritizing productive spending does not translate into immediate fiscal improvements and may instead coincide with looser fiscal positions.

The findings suggest that expenditure composition matters for debt sustainability, but its effects are asymmetric across the fiscal cycle: protecting productive investment during consolidation appears beneficial, whereas prioritization during expansion does not necessarily strengthen fiscal discipline or debt outcomes. The inclusion of the interaction terms associated with λ_7 and λ_8 leads to modest changes in the estimated coefficients of the Iterative Seemingly Unrelated Regression estimation. However, these adjustments are relatively small in magnitude, and the overall signs and economic interpretations of the key parameters remain consistent. This indicates that the core relationships identified in the baseline specification are robust to the inclusion of expenditure-composition effects, and that the underlying conclusions regarding fiscal behaviour and debt sustainability continue to hold.

System: SYSOIL
Estimation Method: Iterative Seemingly Unrelated Regression
Date: 03/19/26 Time: 10:15
Sample: 1982 2025
Included observations: 44
Total system (balanced) observations 264
Simultaneous weighting matrix & coefficient iteration
Convergence achieved after: 13 weight matrices, 14 total coef iterations

		Coefficient	Std. Error	t-Statistic	Prob.
C(10)	α_0	0.005263	0.003325	1.582842	0.1148
C(11)	α_1	0.83605	0.069044	12.10892	0
C(100)	β_0	0.026935	0.040051	0.672516	0.5019
C(101)	β_1	0.129862	0.148645	0.873638	0.3832
C(102)	β_2	-1.60186	0.740588	-2.16296	0.0315
C(20)	γ_0	0.038135	0.014611	2.610072	0.0096
C(21)	γ_1	0.3478	0.130989	2.655189	0.0085

C(26)	γ_2	0.159972	0.064028	2.498474	0.0131
C(30)	δ_0	0.012921	0.008322	1.552654	0.1218
C(32)	δ_1	0.052313	0.142941	0.365979	0.7147
C(33)	δ_2	-0.72486	0.173572	-4.17615	0
C(35)	δ_3	-0.03446	0.022114	-1.55805	0.1205
C(37)	δ_4	0.949243	0.21328	4.450685	0
C(40)	θ_0	0.098886	0.043904	2.252334	0.0252
C(41)	θ_1	-1.31544	0.870328	-1.51142	0.132
C(42)	θ_2	0.567217	0.254836	2.225815	0.027
C(43)	θ_3	-0.87356	0.345222	-2.53043	0.012
C(50)	λ_0	0.000314	0.005116	0.061337	0.9511
C(51)	λ_1	-0.12961	0.057623	-2.24926	0.0254
C(52)	λ_2	0.531896	0.065421	8.130381	0
C(53)	λ_3	-0.05148	0.025185	-2.04389	0.0421
C(54)	λ_4	0.021024	0.007386	2.846504	0.0048
C(55)	λ_5	-0.39099	0.088542	-4.41582	0
C(56)	λ_6	-0.00688	0.003371	-2.04118	0.0423
C(57)	λ_7	0.008786	0.004196	2.093973	0.0373
C(58)	λ_8	-0.00739	0.005192	-1.42296	0.1561
Determinant residual covariance			**2.82E-18**		

Equation: IUS_R = C(10) + C(11) * IUS_R(-1)			
Observations: 44			
R-squared	0.755417	Mean dependent var	0.040074
Adjusted R-squared	0.749593	S.D. dependent var	0.02266
S.E. of regression	0.011339	Sum squared resid	0.0054
Durbin-Watson stat	1.848015		

Equation: OIL = C(100) + C(101) *OIL(-1) +C(102)*OIL(-1)*COVID			
Observations: 44			
R-squared	0.077382	Mean dependent var	0.049238
Adjusted R-squared	0.032377	S.D. dependent var	0.267674
S.E. of regression	0.263306	Sum squared resid	2.842524
Durbin-Watson stat	2.069834		

Equation: IKEN_R = C(20) + C(21) * IKEN_R(-1) + C(26) * XR(-1)			
Observations: 44			
R-squared	0.199416	Mean dependent var	0.075855
Adjusted R-squared	0.160363	S.D. dependent var	0.064871
S.E. of regression	0.059442	Sum squared resid	0.144868
Durbin-Watson stat	2.134549		

Equation: GDP = C(30) + C(32) * GDP(-1) + C(33) * GDP(-1)*DUM + C(35) * XR(-1) + C(37)*EXP_DEV(-1)

Observations: 44			
R-squared	0.223509	Mean dependent var	0.038432
Adjusted R-squared	0.143869	S.D. dependent var	0.022455
S.E. of regression	0.020777	Sum squared resid	0.016835
Durbin-Watson stat	2.168521		

Equation: XR = C(40) + C(41) * GDP(-1) + C(42) * XR(-1) + C(43)*XR(-1)^2			
Observations: 44			
R-squared	0.177067	Mean dependent var	0.069663
Adjusted R-squared	0.115348	S.D. dependent var	0.139493
S.E. of regression	0.131201	Sum squared resid	0.688551
Durbin-Watson stat	2.179092		

Equation: PB = C(50) + C(51)*GAP(-1) *DUM + C(52) * PB(-1)+ C(53) * XR(-1) *DUM + C(54) * DEBT(-1) + C(55)*EXP_DEV_SH(-1) + C(56) *OIL(-1) + C(57) * DEBT(-1) *PROT + C(58) * DEBT(-1) *PRIOR			
Observations: 44			
R-squared	0.93185	Mean dependent var	-0.00709
Adjusted R-squared	0.916273	S.D. dependent var	0.022505
S.E. of regression	0.006512	Sum squared resid	0.001484
Durbin-Watson stat	1.653877		

Table 3: Iterative Seemingly Unrelated Regression Estimation Results Including Expenditure Composition Effects

Figure 3: Fan Chart of Debt to GDP (Baseline)

Fan charts are modelled using DevTech's platform for Dynamic Stochastic Debt Sustainability Analysis. The updated fan chart for the debt-to-GDP ratio remains largely consistent with the baseline results, showing a gradual declining trend with the median falling to approximately 63% over the five-year projection horizon. The distribution continues to widen over time and remains skewed toward the downside, indicating a greater dispersion toward lower debt outcomes. Overall, the similarity in patterns suggests that the underlying debt dynamics are robust to the changes of specification to test H2 and H3, with only minor changes in magnitude and no substantive shift in the overall risk profile. The same is true for the fan chats produced for all estimated variables.

Figure 4: Fan Chart of GDP Growth (Baseline)

The fan chart for GDP growth shows a gradual upward adjustment from recent levels, with the median increasing in 2026 and remaining relatively steady at approximately 5.75% by the end of the projection horizon. The distribution widens over time and is slightly skewed, with a somewhat larger dispersion on the downside in the early years and more balanced risks thereafter.

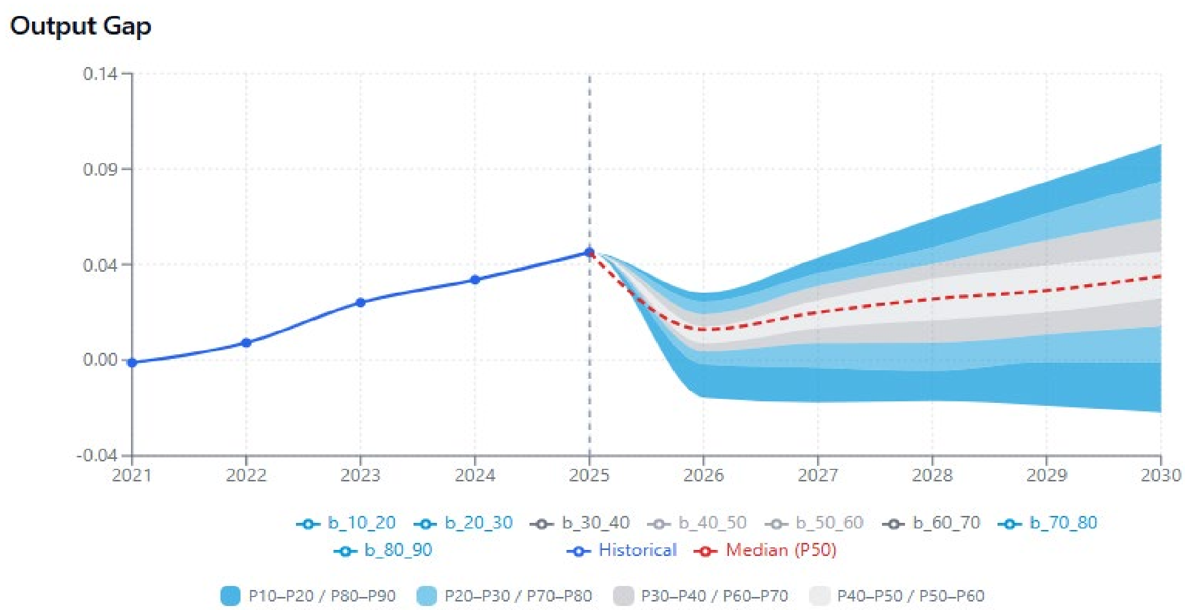

Figure 5: Fan Chart of Output Gap (Baseline)

The fan chart for the output gap shows an initial decline from its peak, followed by a steady recovery, with the median increasing, reaching approximately 3.9% by the end of the projection horizon. The distribution widens over time and is somewhat skewed, with greater downside dispersion in the near term and more balanced risks in later periods. This suggests that while economic activity is expected to strengthen relative to potential output, short-term vulnerabilities remain.

Figure 6: Fan Chart of Primary Balance (Baseline)

The fan chart for the primary balance shows a similar median path compared with recent deficit levels, followed by a mild gradual deterioration. The distribution widens over time and is somewhat skewed to the upside, indicating a greater likelihood of stronger fiscal outcomes relative to weaker ones.

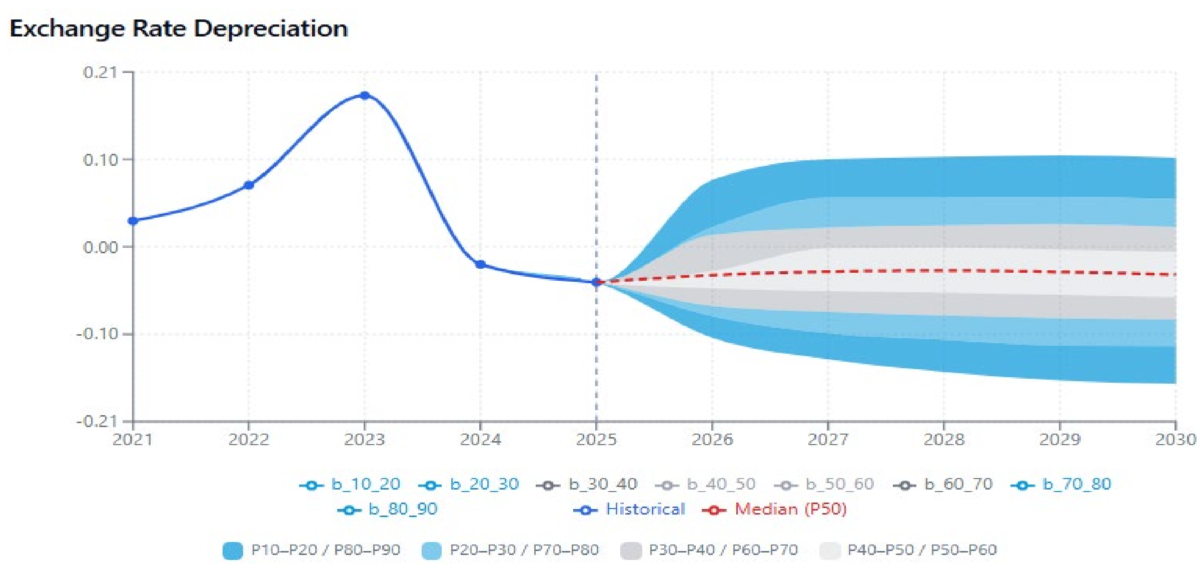

Figure 7: Fan Chart of Real Exchange Rate Change (Baseline)

The fan chart for exchange rate depreciation shows a relatively stable median tendency toward appreciation of around 4%. The distribution is relatively wide, suggesting significant uncertainty around exchange rate dynamics.

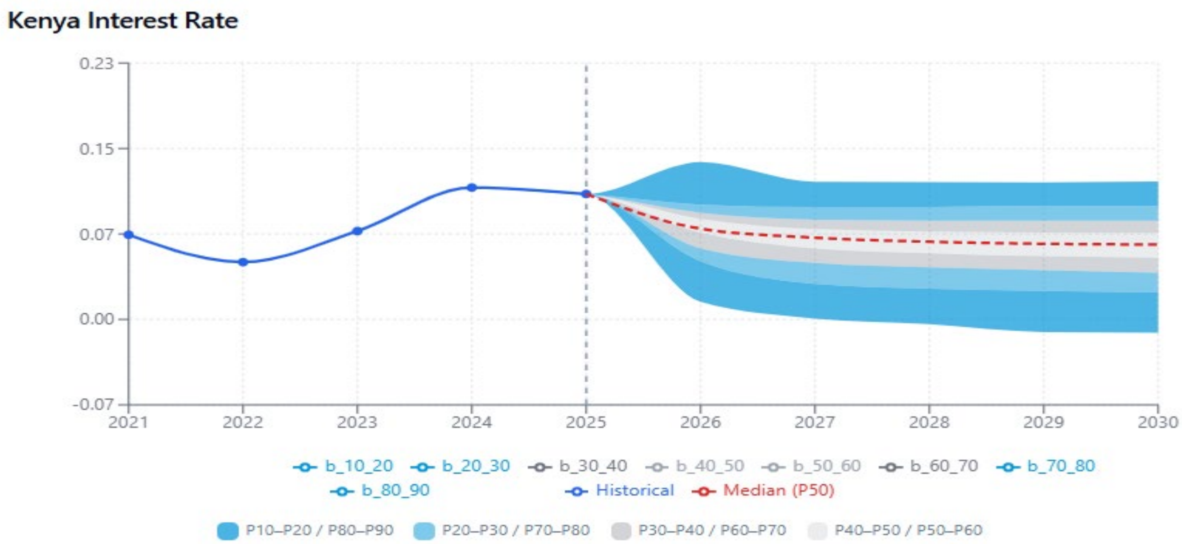

Figure 8: Fan Chart of Real Domestic Interest Rate (Baseline)

The fan chart for Kenya's real interest rate shows a gradual decline from recent elevated levels, with the median decreasing and stabilizing around 6.6% over the projection horizon. The distribution widens over time and is somewhat skewed to the downside, indicating a greater likelihood of lower interest rates than higher ones. This suggests that while borrowing costs are expected to ease, there remains uncertainty driven by domestic macroeconomic conditions and external financial factors.

Foreign Fundamental Forecasts – Baseline Scenario

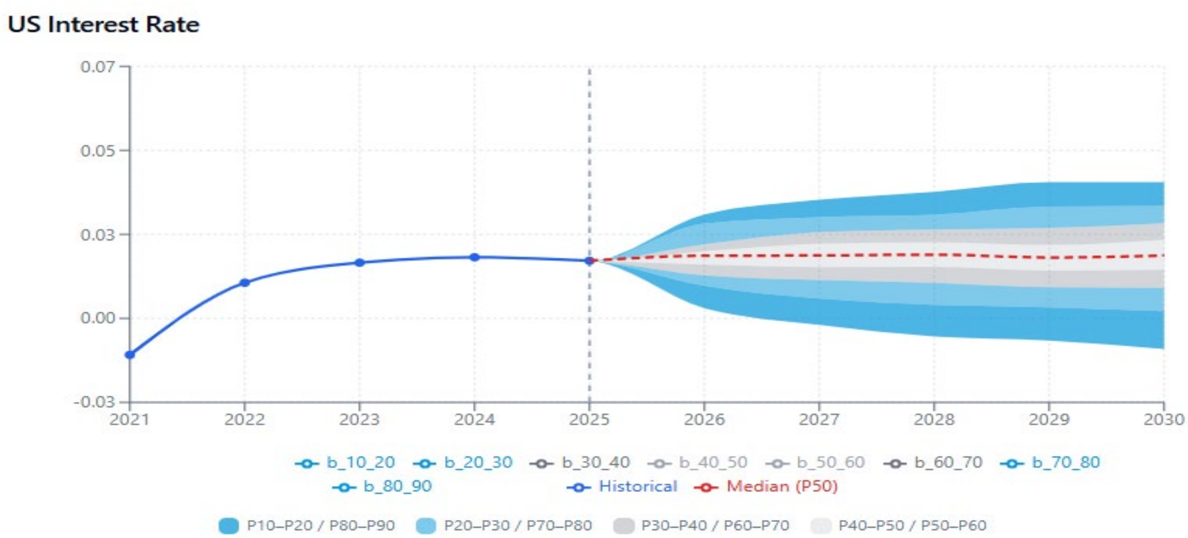

Figure 9: Fan Chart of Global Interest Rates (Proxy US Interest Rates) (Baseline)

The fan chart for the U.S. interest rate shows a relatively stable median path around 1.8%, with only a slight upward adjustment over the projection horizon. The distribution widens gradually and appears symmetric, indicating balanced uncertainty. Something important to highlight is that starting in 2027, the 10th percentile contemplates negatives real interest rates.

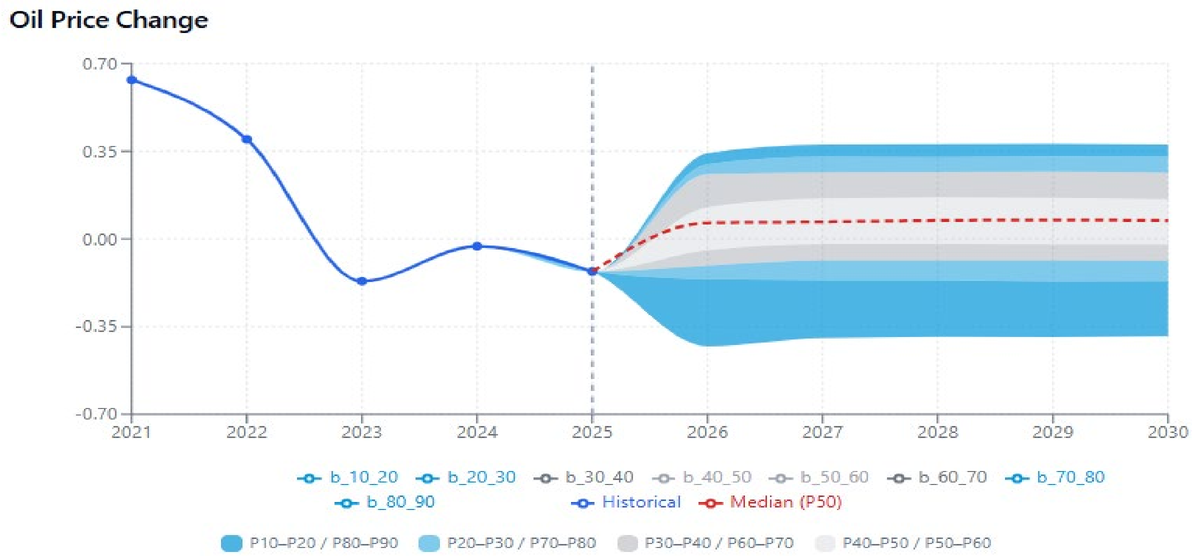

Figure 10: Fan Chart of Commodity Prices (Proxy Oil Prices) (Baseline)

The fan chart for oil price changes shows an expected price recovery from recent declines, with the median change turning positive and increasing gradually over the projection horizon. The distribution widens significantly over time and is somewhat skewed, with substantial dispersion on both the upside and downside. Also notice that current events related to the war in Iran are consistent with the values around the 90th percentile.

PROBABILITIES OF DEBT TO GDP LEVELS (BASELINE)

The analysis allows for assessing the probability that Kenya's debt-to GDP ratio will exceed its official target. To assess fiscal risk relative to policy-relevant benchmarks, the model computes the probability that the debt-to-GDP ratio exceeds specified thresholds over the projection horizon. In the case of Kenya, this analysis is anchored around the official debt target of 55% of GDP (IMF, 2024). Using the full set of Monte Carlo simulation outputs, the framework calculates probabilities of the form $P(debt_t > \overline{d})$ for a range of debt thresholds. These probabilities are derived by evaluating, for each simulation and each projection year, whether the debt ratio exceeds a given threshold, and then aggregating across all simulated paths. The analysis is conducted annually and reported across intervals of 5 percentage points, providing a detailed grid of risk estimates. This approach yields a forward-looking and quantitative assessment of the likelihood that debt levels surpass critical sustainability thresholds, offering a comprehensive view of fiscal risk under uncertainty.

Probability Debt/GDP is higher than	2026	2027	2028	2029	2030
0%	100.00%	100.00%	100.00%	99.90%	99.30%
5%	100.00%	100.00%	100.00%	99.80%	98.70%
10%	100.00%	100.00%	100.00%	99.70%	97.90%
15%	100.00%	100.00%	100.00%	99.50%	97.20%
20%	100.00%	100.00%	100.00%	99.40%	96.20%
25%	100.00%	100.00%	100.00%	98.60%	95.10%
30%	100.00%	100.00%	100.00%	97.90%	93.80%
35%	100.00%	100.00%	99.80%	96.40%	90.90%
40%	100.00%	100.00%	98.90%	94.20%	87.30%
45%	100.00%	100.00%	97.20%	90.00%	82.60%
50%	100.00%	99.90%	92.30%	83.70%	76.20%
55%	100.00%	96.20%	84.40%	74.80%	69.30%
60%	99.40%	85.10%	70.10%	64.20%	56.80%
65%	76.20%	55.40%	49.40%	46.70%	43.50%
70%	20.60%	25.10%	28.20%	30.10%	29.80%
75%	0.00%	5.80%	12.50%	17.90%	20.20%
80%	0.00%	0.20%	5.20%	9.30%	12.60%
85%	0.00%	0.10%	1.10%	4.10%	7.60%
90%	0.00%	0.00%	0.20%	1.50%	4.20%
95%	0.00%	0.00%	0.10%	0.50%	2.30%
100%	0.00%	0.00%	0.00%	0.10%	0.80%

Note: The table in the lower panel shows the probability that the debt / GDP ratio is greater than a specific value of the 0% to 100% range for each year. The graph in the upper shows these distributions for selected years.

Table 4: Probability of Deb to GDP higher than (Baseline)

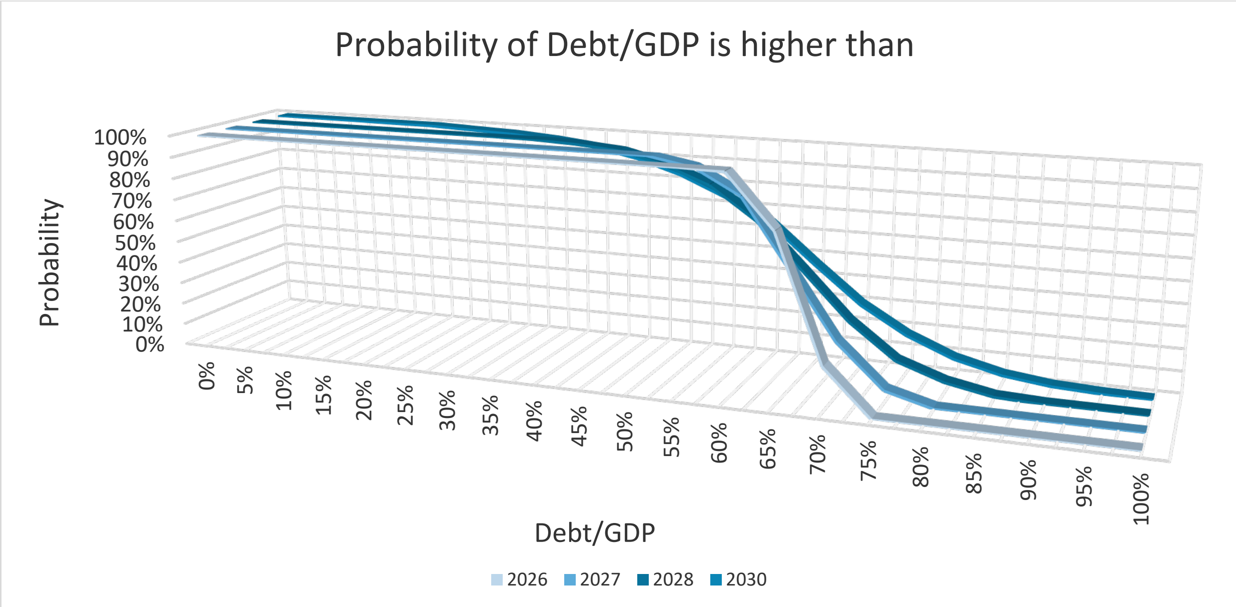

Figure 11: Probability of Deb to GDP higher than (Baseline)

The results indicate a persistently high probability that Kenya's debt-to-GDP ratio remains above moderate thresholds throughout the projection horizon, while risks at higher thresholds increase over time. For instance, the probability that debt is at Kenya's 55% target is 100% in 2026 and declines gradually to 69.3% by 2030. While this downward trend suggests some improvement in debt dynamics, the likelihood of breaching the policy target remains substantial over the entire period.

CONDITIONAL FORECASTS

DevTech's platform for Dynamic Stochastic Debt Sustainability Analysis allows for a fully consistent set of conditional forecasting for scenario building. Beyond the unconditional baseline, the framework generates conditional forecasts based on selected macroeconomic shocks. These are constructed through a shock-bootstrapping procedure, in which specific percentiles from the baseline distribution are used to simulate alternative scenarios. By selecting shocks at different distribution points, the model enables the evaluation of debt dynamics under both likely and extreme conditions. All simulations are anchored to the 50th percentile (median) of the baseline, ensuring consistency across scenarios. For each scenario, a target variable and corresponding conditions are specified, and the simulations are constrained to satisfy these conditions, enhancing the consistency, precision, and interpretability of the results. The following scenarios show the usefulness of the tool to illustrate important macroeconomic risks to debt sustainability.

Lower Economic Growth

Results show that lower economic growth will make debt less sustainable. For illustration purposes, of the following figures show outcomes for which Kenya's economic growth are restricted to remain within a specified range of 3% to 6%. From the full set of simulations, 190 out of 10,000 paths (1.9%) satisfy these conditions and are selected to generate the conditional forecast. Compared to the unconditional distribution, the conditioned fan chart is narrower and centred at higher debt levels, with the median path increasing more rapidly over the projection horizon. This indicates that, under scenarios consistent with moderate and controlled debt levels, debt dynamics deteriorates relative to the baseline (the median value by 2030 increases from 63% of GDP to 69% of GDP).

Figure 12: Fan Chart of GDP Growth (Conditional)

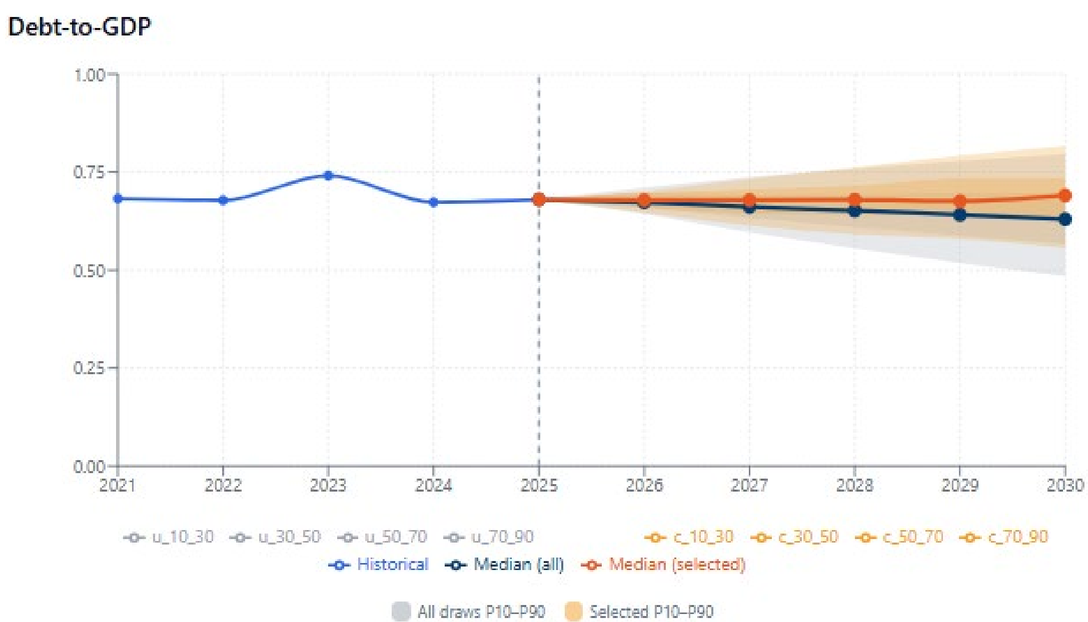

Figure 13: Fan Chart of Debt to GDP (Conditional)

Higher Oil Prices

Results show that higher oil prices growth will make debt to be less sustainable. The following figures show outcomes for which oil prices growth is restricted to remain within a specified range of 25% to 40%. From the full set of simulations, only 9 out of 10,000 paths (0.1%) satisfy these conditions and are selected to generate the conditional forecast. Compared to the unconditional distribution, the conditioned fan chart is narrower and centred at a higher debt-to-GDP ratio, with the median path increasing towards the end of the projection horizon (the median value by 2030 increases from 63% of GDP to 76.8% of GDP). This indicates that the current disruptions in oil supply and the resulting price increases, if sustained, could apply pressure to Kenya's debt sustainability.

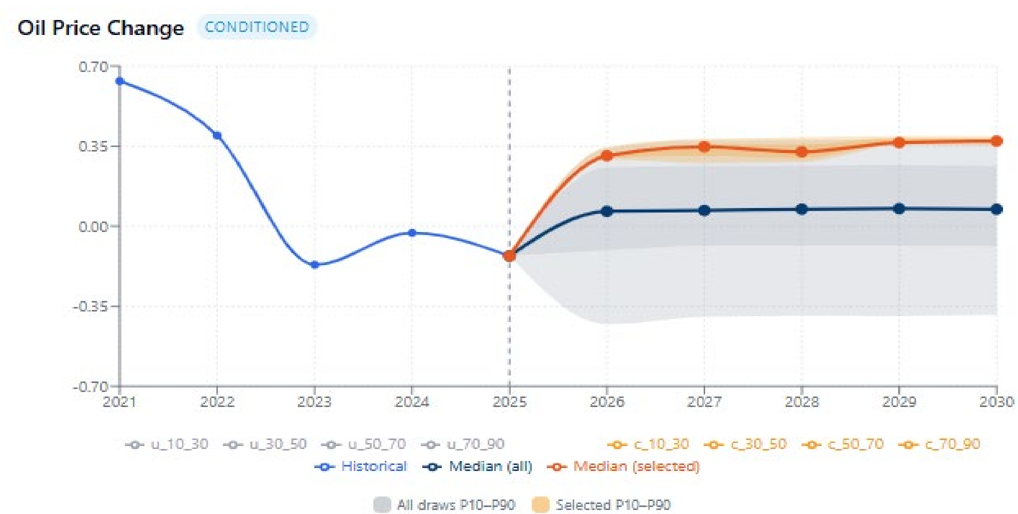

Figure 14: Fan Chart of Oil Prices Growth (Conditional)

Figure 15: Fan Chart of Debt to GDP (Conditional)

Higher Exchange Rate Depreciation

Results show that higher oil prices growth will make debt to be less sustainable. The following figures show outcomes for which exchange rate depreciations are restricted to remain within a specified range of -1% to 8%. From the full set of simulations, only 36 out of 10,000 paths (0.4%) satisfy these conditions and are selected to generate the conditional forecast. Compared to the unconditional distribution, the conditioned fan chart is narrower and centred at a higher debt-to-GDP ratio, with the median path increasing towards the end of the projection horizon (the median value by 2030 increases from 63% of GDP to 70.8% of GDP). This indicates that depreciations of the Kenyan Shilling could bring pressure on its debt sustainability.

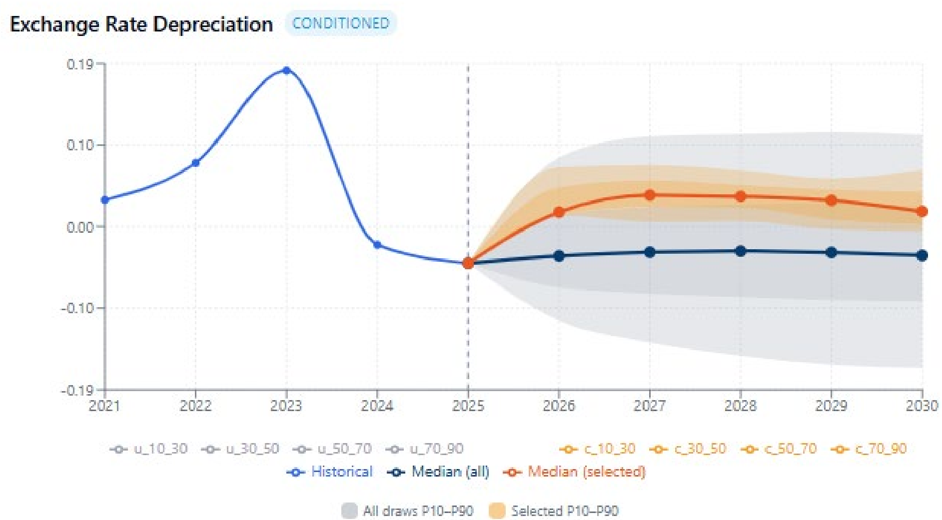

Figure 16: Fan Chart of Exchange Rate Depreciation (Conditional)

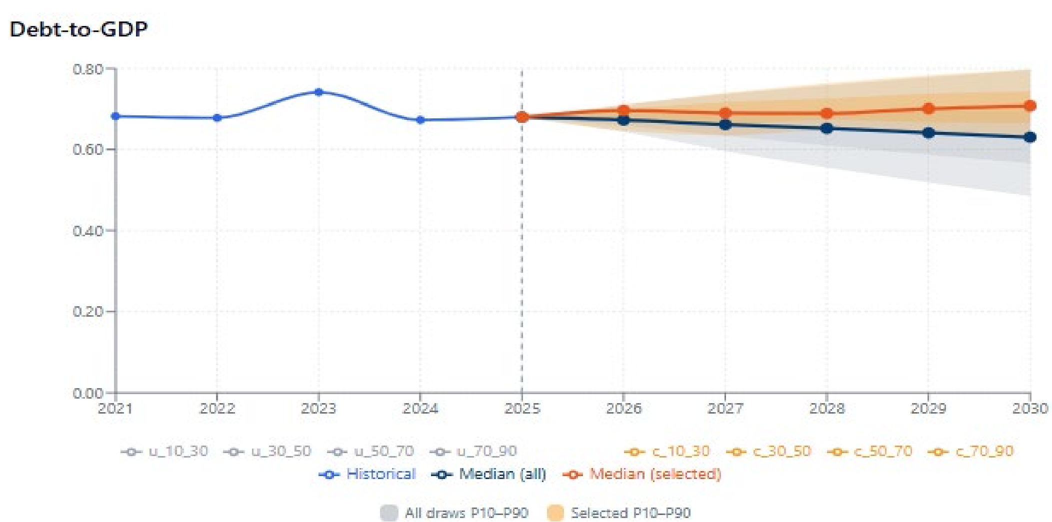

Figure 17: Fan Chart of Debt to GDP (Conditional)

CONCLUSION

This paper develops and applies a stochastic debt sustainability framework that moves beyond traditional deterministic approaches by explicitly incorporating macroeconomic uncertainty, endogenous fiscal policy, and dynamic feedback effects. By jointly modelling fiscal and macroeconomic variables within a unified system and generating probabilistic debt projections through Monte Carlo simulations, the framework provides a more comprehensive and policy-relevant assessment of debt dynamics in uncertain environments. In addition, it provides a fully consistent framework for conditional forecasting that fully considers uncertainty and the internal macroeconomic consistency of the economy.

The empirical application of this framework to Kenya highlights several key findings. First, fiscal policy exhibits a debt-stabilizing response, although it remains procyclical and sensitive to external shocks such as exchange rate movements and commodity price fluctuations. Second, development expenditures play a critical role in shaping debt dynamics through a productive transformation channel, where higher investment supports growth and improves debt sustainability despite short-term fiscal costs. Third, the composition of expenditure matters, particularly during fiscal consolidation, where protecting productive spending strengthens fiscal responses and enhances sustainability outcomes.

The analysis further reveals that, while debt is projected to decline gradually over the medium term, macroeconomic risks remain elevated. There is a persistent probability of exceeding key policy thresholds, including Kenya's 55% debt target. Conditional forecasts demonstrate that debt sustainability can be undermine by critical macroeconomic shocks, underscoring the importance of disciplined fiscal management and growth-enhancing strategies to enhance resilience to negative shocks and its negatives effects on debt sustainability dynamics.

The results emphasize that debt sustainability in African economies is a function of how public productive transformation resources are allocated during fiscal consolidations. Policies that support productive transformation, strengthen fiscal responsiveness, and mitigate exposure to external shocks are central to achieving sustainable debt paths. The framework provides a flexible and scalable tool for policymakers to assess fiscal risks, design robust strategies, and evaluate scenarios and policy trade-offs under uncertainty.

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ANNEXES

DATA SOURCES

Table 5: Variables, Descriptions, and Sources

Variable	Description	Source
Fiscal Variables		
Primary Balance (pb)	Primary balance. Treated as endogenous.	**1981-2025:** WEO Oct. 2025.
Debt-to-GDP Ratio (debt)	Debt as a percentage of GDP. Treated as endogenous.	**1981-2002:** IMF Historic Public Debt Database, 2015. **2003-2024:** Central Bank of Kenya, 2025. **2025:** WEO Oct. 2025.
Development Expenditures (exp_dev)	Development expenditures as a percentage of GDP. Treated as exogenous. Treated as endogenous.	**1981:** Calculated using percentage increase in expenditures from Kenya Central Bureau of Statistics 1982 Economic Survey and 1982 expenditure of general government as a percent of GDP from WEO Oct. 2025. **1982-2000:** Calculated using share of expenditure from Murgo 2017 and expenditure of general government as a percentage of GDP from WEO Oct. 2025. **2001-2024:** Calculated using share of expenditure from Central Bank of Kenya, 2025 and expenditure of general government as a percentage of GDP from WEO Oct. 2025. **2025:** Calculated using share of expenditure from Kenya 2025/2026 Budget Brief and expenditure of general government as a percentage of GDP from WEO Oct. 2025.
Development Expenditures (exp_dev_sh)	Development expenditures as a percentage of total expenditures. Treated as exogenous.	**1981-2000:** Calculated using share of expenditure from Murgo, 2017. **2001-2024:** Calculated using share of expenditure from Central Bank of Kenya, 2025. **2025:** Calculated using share of expenditure from Kenya 2025/2026 Budget Brief.
Share of External Debt (dus)	Share of external debt.	**1981-1998:** **1999-2025:** Calculated from Central Bank of Kenya, 2025 data. 2025 data calculated using 2025 April data (latest data point available at time of study).
Structural Dummy (dum)	Structural dummy variable included to control for periods of high primary deficits.	**1981-2025:** Calculated using WEO Oct. 2025 data.
Macroeconomic Variables		
GDP Growth (gpd)	Real GDP growth. Treated as endogenous.	**1981-2025:** WEO Oct. 2025.
Kenya Real Interest Rate (iken_r)	Kenya real interest rate. Treated as endogenous.	**1981-2023:** World Bank 2025. **2024-2025:** Central Bank of Kenya 2025.

Real Exchange Rate (xr)	Nominal exchange rate growth. Treated as endogenous.	**1981-2025:** WEO Oct. 2025.
Output Gap (gap)	Output gap (HP-filter deviation from trend GDP). Treated as endogenous.	**1981-2025:** Calculated using WEO Oct. 2025 data.
External Variables		
US Real Interest Rate (ius_r)	Global interest rates proxied by U.S. rates. Treated as exogenous.	**1981-2021:** World Bank 2025. **2023-2025:** FRED 2025.
Oil Price Growth (oil)	Commodity prices proxied by oil price growth. Treated as exogenous.	**1981-2025:** WEO Oct. 2025.
COVID-19 (covid)	Captures the impact of COVID-19 on the global economy.	Calculated.

DEBT SUSTAINABILITY FOR THE "SYNTHETIC" AFRICAN ECONOMY: A Time Series Approach

DATA CONSTRUCTION

We use the same sample as the OECD team to create a "synthetic" African economy. For that we use a panel of countries from 1990 to 2019 for which there is data on debt as share of GDP, primary balance as share of GDP, nominal exchange rate with respect to the US dollar, public gross capital formation as share of GDP, domestic real interest rates, and real GDP growth. This set of indicators is complemented by the change in nominal oil prices and the US real interest rates. The "synthetic" African economy is constructed as the weighted average (using the GDP in USD) of all values across countries for a given year. Changes in the nominal exchange rate proved to be one of the most volatile variables in the entire sample, which motivate the drop of several countries from the data creation process, either because they presented extreme values (Somalia, South Sudan, Zambia, and Zimbabwe) or because they belong to exchange rate agreements that keep the currency fixed, eliminating the validity of the exchange rate mechanism (Benin, Burkina Faso, Côte d'Ivoire, Guinea-Bissau, Mali, Niger, Senegal, Togo, Cameroon, Chad, Congo Republic, Equatorial Guinea, Central African Republic, Gabon, Cabo Verde, São Tomé and Príncipe, Namibia, Lesotho, Eswatini, Comoros, and Djibouti). Note that they represent in 2024 11.3% of total Continental GDP.[2]

ESTIMATION RESULTS

This section presents the results for the same modeling and estimation strategy for the Kenya case, but using the "synthetic" African economy. Here are all the relevant set of equations for the "Synthetic" African Economy.

The fiscal reaction function is:

$$\boldsymbol{pb_t} = \lambda_0 + \lambda_1 gdp_{t-1} + \lambda_2 pb_{t-1} + \lambda_3 debt_{t-1} + \lambda_4 (debt_{t-1} \cdot prot_t) + \lambda_5 (debt_{t-1} \cdot prior_t) + \lambda_5 dum2000_t$$

λ_4>0 supports H2

λ_5>0 supports H3

Where:

prot = 1 if $Consolidation_t = 1$ and $exp\_dev\_sh_t \geq exp\_dev\_sh_{t-1}$

prior = 1 if $Expansion_t = 1$ and $exp\_dev\_sh_t > exp\_dev\_sh_{t-1}$

exp_dev_sh = the development expenditure share of total expenditure.

The macroeconomic system is specified as follows:

[2] Also notice that only 2 countries have debt and primary balance data since 1990.

$$\boldsymbol{ius\_r_t} = \alpha_0 + \alpha_1 ius\_r_{t-1}$$

$$\boldsymbol{oil_t} = \beta_0 + \beta_1 oil_{t-1} + \beta_2 oil_{t-2}$$

$$\boldsymbol{iafri\_r_t} = \gamma_0 + \gamma_1 iafri\_r_{t-1} + \gamma_2 ius\_r_{t-1} + \gamma_3 dum2000_t$$

$$\boldsymbol{gdp_t} = \delta_0 + \delta_1 gdp_{t-1} + \delta_2 (xr_{t-1} \cdot dum2000_t) + \delta_3 exp\_dev_{t-1} + \delta_4 dum2000_t$$

$$\boldsymbol{xr_t} = \theta_0 + \theta_1 xr_{t-1} + \theta_2 (exp\_dev_{t-1} \cdot dum2000_t) + \theta_3 iafri\_r_{t-1}$$

The evolution of the debt-to-GDP ratio is governed by the following motion equation:

$$\boldsymbol{debt_t} = \left(\frac{(1 + ius\_r_t)(1 + xr_t) \cdot dus_t \cdot debt_{t-1} + (1 + iafri\_r_t)(1 - dus_t) \cdot debt_{t-1}}{1 + gdp_t} \right) - pb_t$$

where *dus* denotes the share of external (foreign currency or dollar-linked) debt in total public debt.

The SUR estimation results highlight several key mechanisms driving debt dynamics, including the role of public investment on productive transformation, in the African context. The evidence supports the presence of a productive transformation channel; whereby higher development expenditures are associated with increased real GDP growth (δ_3 = 0.837). This finding suggests that public investment plays a critical role in enhancing economic performance, which in turn can improve debt sustainability through higher growth.

The results also confirm the importance of endogenous fiscal policy responses, with some of them serving as debt stabilizers. Higher levels of previous public debt are associated with improvements in the primary balance (λ_3 = 0.059), indicating that fiscal authorities tend to have less expansionary fiscal policy when experiencing rising debt burdens. At the same time, fiscal policy appears to be procyclical (λ_1 = 0.309), with fiscal balances moving in tandem with the business cycle rather than stabilizing it.

Taken together, these results underscore the complex interaction between fiscal policy, macroeconomic conditions, and external shocks. They highlight the dual role of fiscal policy as both a response mechanism to debt accumulation and a transmission channel through which macroeconomic shocks influence debt sustainability. The following table shows the results of the joint estimation of the fiscal policy reaction function and the estimation by iterative seemingly unrelated regression. Notice that the evidence is found to support the existence of this feedback channel. Notice that given that the data ends in 2019, all projections should be interpreted as changes with respect to a generic t time. So, our forecasting sample can be framing to be from t+1 to t+5.

System: SYSOIL
Estimation Method: Iterative Seemingly Unrelated Regression
Date: 05/19/26   Time: 11:54
Sample: 1990 2019
Included observations: 30
Total system (unbalanced) observations 179
Simultaneous weighting matrix & coefficient iteration
Convergence achieved after: 48 weight matrices, 49 total coef iterations

		Coefficient	Std. Error	t-Statistic	Prob.
C(10)	α_0	0.00756	0.003761	2.010197	0.0462
C(11)	α_1	0.780219	0.084804	9.200235	0
C(100)	β_0	0.086096	0.043184	1.993701	0.048
C(101)	β_1	-0.00831	0.110065	-0.07549	0.9399
C(102)	β_2	-0.35902	0.098886	-3.63066	0.0004
C(20)	γ_0	-0.02223	0.026058	-0.85303	0.395
C(21)	γ_1	0.248609	0.139475	1.782462	0.0766
C(27)	γ_2	0.832315	0.412355	2.018443	0.0453
C(28)	$\gamma 3$	0.031713	0.018325	1.730539	0.0855
C(30)	δ_0	-0.0299	0.015132	-1.97584	0.05
C(32)	δ_1	-0.39803	0.151868	-2.62093	0.0096
C(34)	δ_2	-0.05751	0.015575	-3.69241	0.0003
C(37)	δ_3	0.837406	0.201802	4.149634	0.0001
C(38)	δ_4	0.049021	0.007142	6.863602	0
C(40)	θ_0	0.767405	0.16431	4.670475	0
C(42)	θ_1	-0.13821	0.185497	-0.74506	0.4574
C(45)	θ_2	-9.96203	2.329764	-4.27598	0
C(46)	θ_3	-2.06314	1.495162	-1.37987	0.1696
C(50)	λ_0	-0.02872	0.011079	-2.5924	0.0104
C(51)	λ_1	0.309481	0.156666	1.975412	0.05
C(52)	λ_2	0.401546	0.082735	4.853406	0
C(54)	λ_3	0.059532	0.016795	3.544646	0.0005
C(57)	λ_4	0.002813	0.008897	0.316128	0.7523
C(58)	λ_5	-0.01872	0.009868	-1.897	0.0597
C(59)	λ_6	-0.01217	0.005728	-2.12382	0.0353
Determinant residual covariance			**3.86E-18**		

Equation: IUS_R = C(10) + C(11) * IUS_R(-1)			
Observations: 30			
R-squared	0.736037	Mean dependent var	0.038252
Adjusted R-squared	0.72661	S.D. dependent var	0.018885
S.E. of regression	0.009874	Sum squared resid	0.00273
Durbin-Watson stat	1.127863		

Equation: OIL = C(100) + C(101) *OIL(-1) + C(102) *OIL(-2)			
Observations: 30			
R-squared	0.093129	Mean dependent var	0.074606
Adjusted R-squared	0.025953	S.D. dependent var	0.256061
S.E. of regression	0.252716	Sum squared resid	1.72437
Durbin-Watson stat	1.572771		

Equation: IAFRI_R = C(20) + C(21) * IAFRI_R(-1) + C(27) * IUS_R(-1) + C(28) *DUM2000
Observations: 30

R-squared	0.093726	Mean dependent var	0.040568
Adjusted R-squared	-0.010844	S.D. dependent var	0.045392
S.E. of regression	0.045638	Sum squared resid	0.054153
Durbin-Watson stat	1.864362		

Equation: GDP = C(30) + C(32) * GDP(-1) + C(34) * XR(-1) * DUM2000 + C(37) * EXP_DEV(-1) + C(38) * DUM2000			
Observations: 30			
R-squared	0.29773	Mean dependent var	0.037643
Adjusted R-squared	0.185367	S.D. dependent var	0.018164
S.E. of regression	0.016394	Sum squared resid	0.006719
Durbin-Watson stat	0.950569		

Equation: XR = C(40) + C(42) * XR(-1) + C(45) * EXP_DEV (-1) * DUM2000 + C(46) * IAFRI_R			
Observations: 30			
R-squared	0.395288	Mean dependent var	0.250765
Adjusted R-squared	0.325514	S.D. dependent var	0.424211
S.E. of regression	0.348393	Sum squared resid	3.155812
Durbin-Watson stat	1.929428		

Equation: PB = C(50) + C(51)*GDP(-1) + C(52) * PB(-1) +C(54) * DEBT(-1) + C(57) * DEBT(-1) *PROT + C(58) * DEBT(-1) *PRIOR + C(59) * DUM2000			
Observations: 30			
R-squared	0.56568	Mean dependent var	0.00504
Adjusted R-squared	0.44723	S.D. dependent var	0.026702
S.E. of regression	0.019853	Sum squared resid	0.008671
Durbin-Watson stat	1.629017		

Table A.6: Iterative Seemingly Unrelated Regression Estimation Results

PRODUCTIVE TRANSFORMATION EXPENDITURES AND DEBTS SUSTAINABILITY

Variable	Low	Current	High
Debt	80.41	68.03	56.97
Xr	19.01	10.35	1.58
Gdp	3.16	4.13	5.13

Table A.7: Productive Expenditure Scenario Results

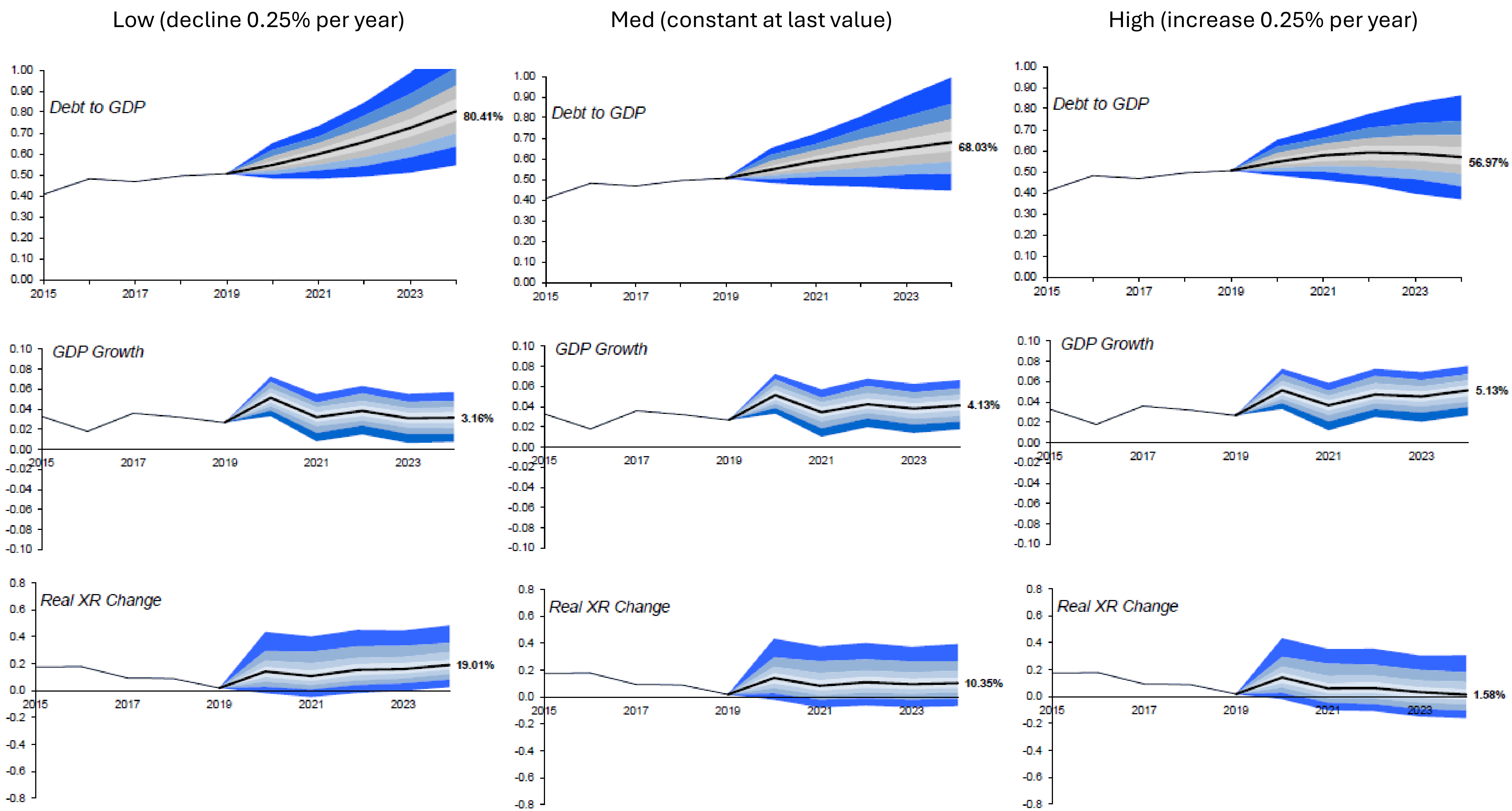


Figure A.18: Fan Chart of Debt to GDP (Baseline)

As mentioned in the methodology, our first hypothesis (H1) tests whether increasing productive transformation expenditure affects the sustainability of public debt in African economies. The results of three productive expenditure scenarios (low, constant, and high) provide strong support for H1, indicating that higher development expenditures contribute to improved debt sustainability through their impact on macroeconomic performance, including higher economic growth.

The three scenarios illustrate how changes in the share of development spending shape both fiscal and economic outcomes. In the low scenario, where development expenditure declines by 0.25 percentage points annually, the economy experiences weaker growth (3.16%) and a higher depreciation of the exchange rate (19.01%). The debt-to-GDP ratio increases to 80.41%, and shows a unsustainable path. This suggests that fiscal consolidation achieved through reduced productive spending may come at the cost of lower growth and higher depreciation of the exchange rate, ultimately worsening debt dynamics. In the constant scenario, development expenditure remains constant, resulting in moderate growth (4.13%) and a lower depreciation of the nominal exchange rate (10.35%). The debt-to-GDP ratio starts stabilizing at 68.03%, but still is not on a stable path. This scenario reflects a continuation of current policies, yielding still unremarkable improvements in debt sustainability. In the high scenario, where development expenditure increases by 0.25 percentage points annually, the economy benefits from significantly stronger growth (5.13%), indicating more robust economic expansion. The exchange rate depreciation is significantly lower (1.58%), with a stable debt-to-GDP ratio declining to 56.97%. This improvement is supported by stronger growth, and a lower depreciating exchange rate, all of which contribute to more favorable debt dynamics.

As exp_dev ↑ ⇒ debt ↓

⇒ gdp ↑

⇒ xr ↓

Figure 19: Productive Transformation Effects

FISCAL ALLOCATION, PRODUCTIVE EXPENDITURE AND DEBTS SUSTAINABILITY

This section evaluates whether the composition of public expenditure influences the sustainability of public debt in African economies, specifically testing H2 (consolidation) and H3 (expansion). The results provide mixed evidence on whether the composition of public expenditure affects debt sustainability through fiscal behavior. The positive coefficient on the interaction term between lagged debt and the Protect indicator (λ_4 = 0.002813) is not statistically significant, so there is no support to H2 suggesting that during periods of fiscal consolidation, maintaining or increasing the share of development expenditures strengthens the fiscal response to rising debt. In contrast, the negative coefficient on the interaction term with the Prioritize indicator (λ_5 = −0.018719) does not support H3, indicating that increasing the share of development expenditures during expansionary periods weakens the fiscal response to debt. This may reflect that, during expansions, prioritizing productive spending does not translate into immediate fiscal improvements and may instead coincide with looser fiscal positions. The important aspect to highlight is that even when λ_5 is negative, the net coefficient on lagged debt is still positive. So, the general primary balance and debt stabilizing relationship is still present, driven by the direct effect (λ_3 = 0.059532).

CONDITIONAL FORECASTS

Lower Economic Growth

Results show that lower economic growth will make debt less sustainable. For illustration purposes, the following figures show outcomes for which Africa's economic growth is restricted to remain within a specified range of 2% to 3% for years t+1 and t+2. From the full set of simulations, 241 out of 10,000 paths (2.4%) satisfy these conditions and are selected to generate the conditional forecast. Results show higher debt levels, with the median path increasing more rapidly over the projection horizon (the median value by year 5 increases from 68% of GDP to 80% of GDP).

Figure 20: Fan Chart of GDP Growth (Conditional)

Figure 21: Fan Chart of Debt to GDP (Conditional)

Higher Oil Prices

Results show that higher oil prices growth will make debt to be less sustainable. The following figures show outcomes for which oil prices growth is restricted to remain within a specified range of an increase of 30% to 40% in the first year and an increase of 20% to 30% in the second year. From the full set of simulations, only 324 out of 10,000 paths (3.2%) satisfy these conditions and are selected to generate the conditional forecast. Results show a higher debt-to-GDP ratio, with the median path increasing towards the end of the projection horizon (the median value by year 5 increases from 68% of GDP to 71% of GDP). This indicates that the current disruptions in oil supply and the resulting price increases, if sustained, could apply pressure to Africa's debt sustainability.

Figure 22: Fan Chart of Oil Prices Growth (Conditional)

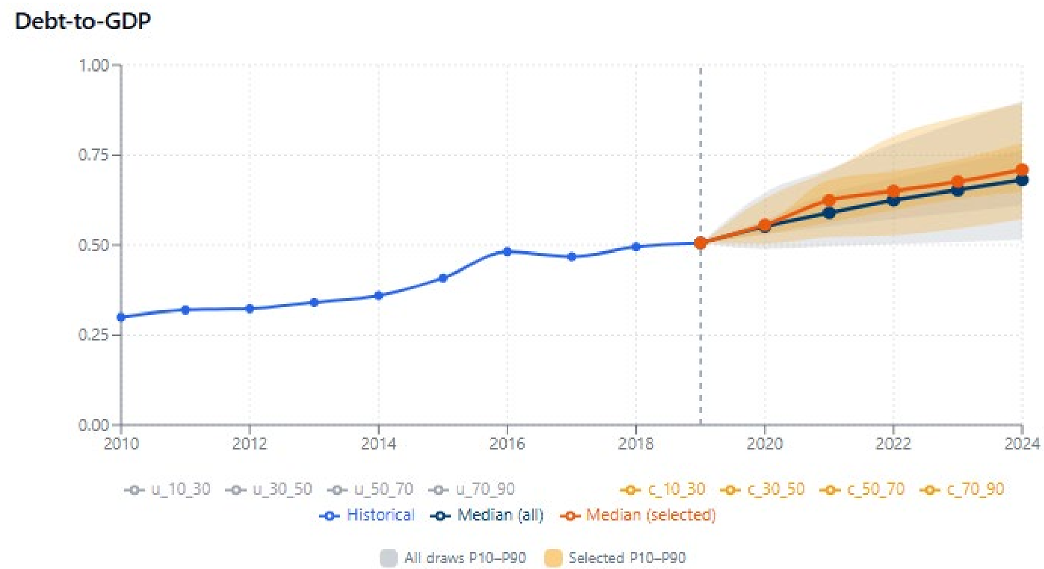


Figure 23: Fan Chart of Debt to GDP (Conditional)

Higher Exchange Rate Depreciation

Results show that higher oil prices growth will make debt to be less sustainable. The following figures show outcomes for which exchange rate depreciations are restricted to remain within a specified range of 15% to 20% for the first two years of forecast. From the full set of simulations, only 31 out of 10,000 paths (0.3%) satisfy these conditions and are selected to generate the conditional forecast. Results show a higher debt-to-GDP ratio, with the median path increasing towards the end of the projection horizon (the median value by 2030 increases from 68% of GDP to 78% of GDP). This indicates that depreciations of the exchange rate could bring significant pressure on Africa's debt sustainability.

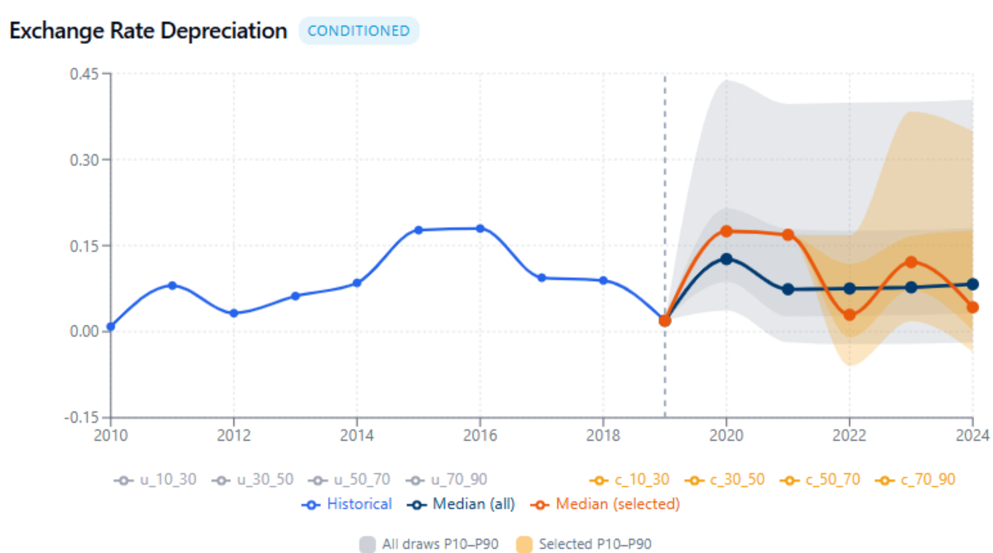


Figure 24: Fan Chart of Exchange Rate Depreciation (Conditional)

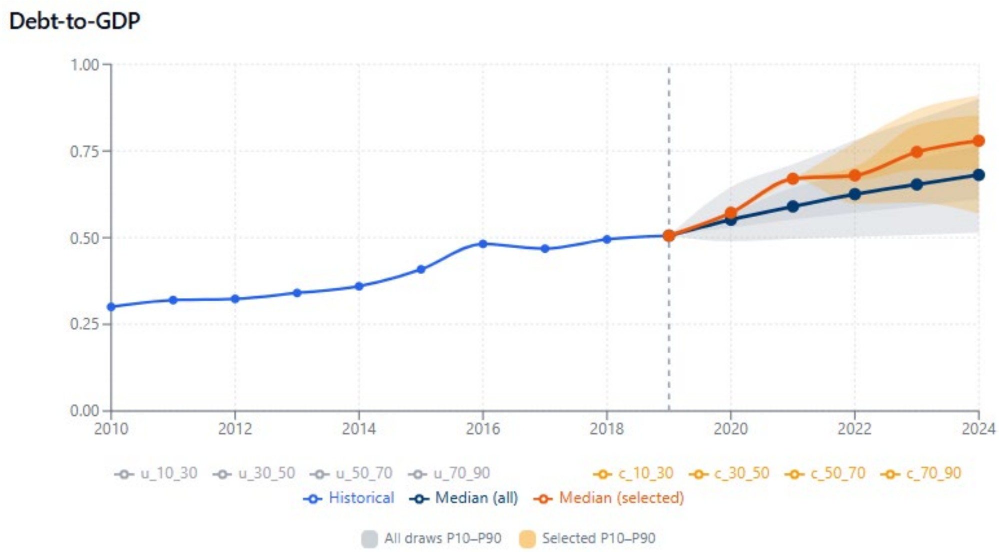


Figure 25: Fan Chart of Debt to GDP (Conditional)

DEBT SUSTAINABILITY FOR A PANEL OF AFRICAN ECONOMIES: Structural (Panel) Vector Autoregressive Analysis Perspective

DATA CONSTRUCTION

We use the same sample as the OECD team used for their estimation of the Primary Balance panel regression. For that we use a panel of countries from 1990 to 2019 for which there is data on debt as share of GDP, primary balance as share of GDP, nominal exchange rate with respect to the US dollar, public gross capital formation as share of GDP, domestic real interest rates, and real GDP growth. This set of indicators is complemented by the change in nominal oil prices and the US real interest rates. The "synthetic" African economy is constructed as the weighted average (using the GDP in USD) of all values across countries for a given year. Changes in the nominal exchange rate proved to be one of the most volatile variables in the entire sample, which motivate the drop of several countries from the data creation process, either because they presented extreme values (Somalia, South Sudan, Zambia, and Zimbabwe) or because they belong to exchange rate agreements that keep the currency fixed, eliminating the validity of the exchange rate mechanism (Benin, Burkina Faso, Côte d'Ivoire, Guinea-Bissau, Mali, Niger, Senegal, Togo, Cameroon, Chad, Congo Republic, Equatorial Guinea, Central African Republic, Gabon, Cabo Verde, São Tomé and Príncipe, Namibia, Lesotho, Eswatini, Comoros, and Djibouti). Note that they represent in 2024 11.3% of total Continental GDP.[3]

[3] Also notice that only 2 countries have debt and primary balance data since 1990.

METHODOLOGY

Our Structural (Panel) Vector Autoregressive Analysis Perspective approach employs a recursive Global–Domestic Panel Factor-VAR to perform a SVAR based Innovation Analysis. The model is estimated equation-by-equation with Fixed Effects. Country-specific fixed effects are removed through dummy variables, ensuring identification from within-country time variation and avoiding bias from correlation between regressors and unobserved heterogeneity. This is used to recover impulse response functions and the error variance decomposition.

To address cross-sectional dependence arising from global co-movements, the model includes two global factors. These are defined as the first two principal components of the cross-country means of all domestic variables. These factors capture pervasive common shocks and mitigate bias from cross-correlated errors, following the logic of common-factor approaches. Note that this is an ideal approach to connect a panel model with global variables when there is interest in estimating the effects of shocks on them.

Model Specification

Domestic variables

$$
\begin{aligned}
X_1 &= \text{PB\_GDP} \\
X_2 &= \text{DEBT\_GDP} \\
X_3 &= \text{GDP\_GROWTH} \\
X_4 &= \text{RIR} \\
X_5 &= \text{XR\_GROWTH} \\
X_6 &= \text{PUB\_GFCF\_GDP}
\end{aligned}
$$

Global variables and common factors

oil = oil real price growth rate

r^{usa} = US representative real interest rate

Z_1 = FACTOR1: first principal component of cross-country means

Z_2 = FACTOR2: second principal component of cross-country means

Panel Factor-VAR Equations:

Global block (Note: *lag of global variables in Factor equations excluded for ease of exposition*)

$$oil_t = \mathrm{b}_{10} + b_{11}oil_{t-1} + b_{12}r_{t-1}^{usa} + u_t^{oil}$$

$$r_t^{usa} = b_{20} + b_{21}oil_{t-1} + b_{22}r_{t-1}^{usa} + u_t^{usa}$$

$$Z_{1t} = \alpha_1 + \gamma_{11}Z_{1,t-1} + \gamma_{12}Z_{2,t-1} + \gamma_{13}oil_t + \gamma_{14}r_t^{usa} + \varepsilon_{1,t}$$

$$Z_{2t} = \alpha_2 + \gamma_{21}Z_{1,t-1} + \gamma_{22}Z_{2,t-1} + \gamma_{23}oil_t + \gamma_{24}r_t^{usa} + \varepsilon_{2,t}$$

Domestic block

Each domestic variable follows (Note: *lag of Factors excluded for ease of exposition*):

$$X_{1,t} = \alpha_3 + \beta_{31}X_{1,t-1} + \beta_{32}X_{2,t-1} + \beta_{33}X_{2,t-1} + \beta_{34}X_{2,t-1} + \beta_{35}X_{2,t-1} + \beta_{36}X_{2,t-1} + \gamma_{31}Z_{1,t} + \gamma_{32}Z_{2,t} + \varepsilon_{3,t}$$

$$X_{2,t} = \alpha_4 + \beta_{41}X_{1,t-1} + \beta_{42}X_{2,t-1} + \beta_{43}X_{2,t-1} + \beta_{44}X_{2,t-1} + \beta_{45}X_{2,t-1} + \beta_{46}X_{2,t-1} + \gamma_{41}Z_{1,t} + \gamma_{42}Z_{2,t} + \varepsilon_{4,t}$$

$$X_{3,t} = \alpha_5 + \beta_{51}X_{1,t-1} + \beta_{52}X_{2,t-1} + \beta_{53}X_{2,t-1} + \beta_{54}X_{2,t-1} + \beta_{55}X_{2,t-1} + \beta_{56}X_{2,t-1} + \gamma_{51}Z_{1,t} + \gamma_{52}Z_{2,t} + \varepsilon_{3,t}$$

$$X_{4,t} = \alpha_6 + \beta_{51}X_{1,t-1} + \beta_{62}X_{2,t-1} + \beta_{63}X_{2,t-1} + \beta_{64}X_{2,t-1} + \beta_{65}X_{2,t-1} + \beta_{66}X_{2,t-1} + \gamma_{61}Z_{1,t} + \gamma_{62}Z_{2,t} + \varepsilon_{3,t}$$

$$X_{5,t} = \alpha_7 + \beta_{71}X_{1,t-1} + \beta_{72}X_{2,t-1} + \beta_{73}X_{2,t-1} + \beta_{74}X_{2,t-1} + \beta_{75}X_{2,t-1} + \beta_{76}X_{2,t-1} + \gamma_{71}Z_{1,t} + \gamma_{72}Z_{2,t} + \varepsilon_{3,t}$$

$$X_{6,t} = \alpha_8 + \beta_{81}X_{1,t-1} + \beta_{82}X_{2,t-1} + \beta_{83}X_{2,t-1} + \beta_{84}X_{2,t-1} + \beta_{85}X_{2,t-1} + \beta_{86}X_{2,t-1} + \gamma_{81}Z_{1,t} + \gamma_{82}Z_{2,t} + \varepsilon_{3,t}$$

Shock Identification

Structural shocks are identified using sign restrictions (Kilian & Lütkepohl, 2017). A standardized public-investment shock is imposed such that:

PUB_GFCF_GDP responds positively for at least five years,

DEBT_GDP responds negatively for at least five years,

GDP_GROWTH responds positively for at least five years.

These sign restrictions identify a public-investment-driven structural innovation consistent with theoretical criteria.[4] However, once the identification is imposed, the estimation is free to determine the sign of the response after the fourth year, as well as the magnitude and statistical significance of the shocks at all horizons. The following section presents the results of the Structural Panel VAR estimation, concentrating on the relevance of the shocks and the magnitude and duration of the effects.

[4] The reduced-form VAR residuals are forecast errors that are generally contemporaneously correlated across equations, whereas the structural shocks are interpreted as economically meaningful innovations that are orthogonal by construction. The structural form imposes identifying restrictions that allow the econometrician to recover these underlying shocks from the reduced-form residuals. In practical terms, this implies imposing restriction on the parameters matrices that relate residuals (epsilon) and structural innovations (u), i.e, matrices C_i in epsilon(t) = C_i . u(t-i) for i=0,1,2,3... There are many values of C_i equally likely given the data, related to the multiple possible ortogonalizations of Sigma (covariance of epsilon); however, in this type of identification strategy only the simulated ortogonalizations that respect the sign restrictions are selected. See Hamilton, James D. (1994), Time Series Analysis, Princeton University Press.

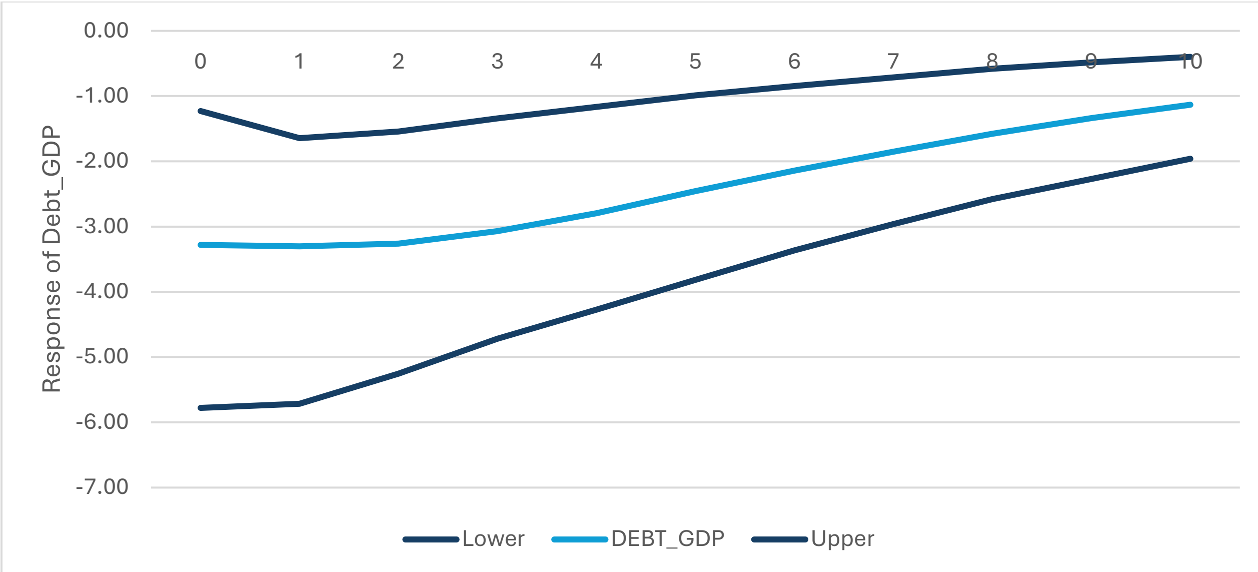
ESTIMATION RESULTS

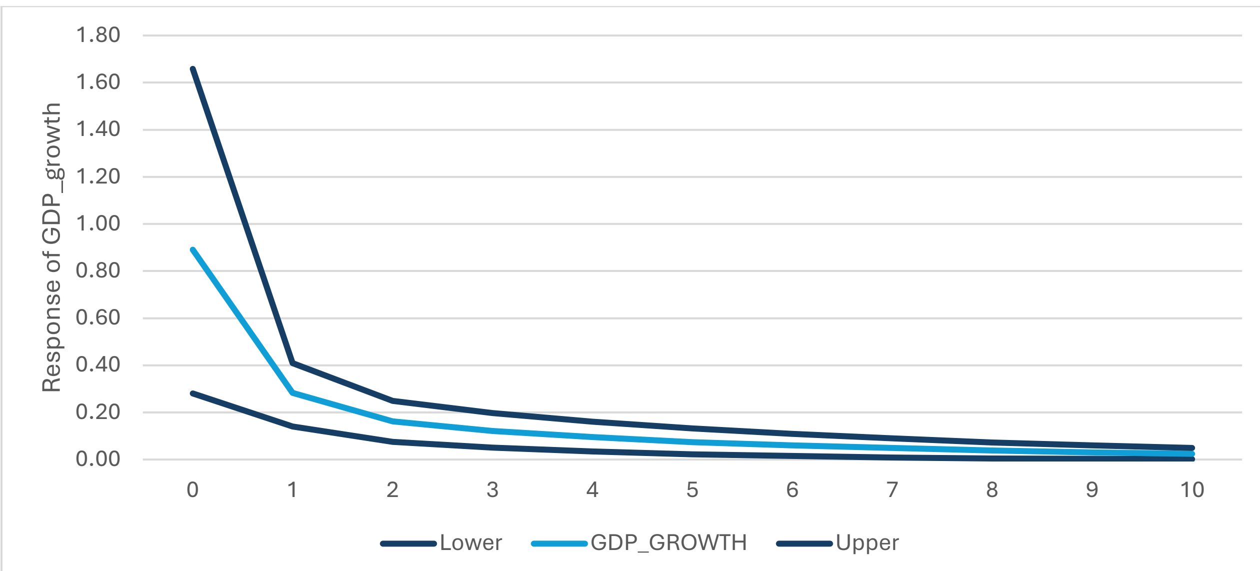
Impulse Response Functions

This section presents the impulse response functions (IRF) capturing the dynamic interactions across the variables in our Panel VAR model. This is a typical tool used in Vector Autoregression (VAR) analysis to examine how economic variables respond over time to an unexpected shock in one variable within the system, in our case to shocks in productive transformation investments. In a VAR framework, variables are assumed to interact dynamically with one another, meaning that changes in one variable can influence the future behavior of other variables.

In policy analysis, impulse response functions are particularly useful because they provide a clear illustration of the dynamic transmission of policy shocks throughout the economy over time. The IRF traces the direction, magnitude, and duration of these effects following a one-time shock. The results help identify whether the effects are positive or negative, how large they are, and how long they persist before returning to equilibrium.

The IRF shows that a typical positive shock to productive transformation investments (half percentage point) contributes to debt sustainability by reducing debt to GDP over three percentage points and one additional percentage point of real GDP growth. These effects are statistically significant and persistent over time, particularly for debt dynamics where even at year 10 after the shock debt tends to be 1 percentage points lower than in the baseline. However, the effects on GDP growth tend to significantly reduce after three years. On impact, a one percentage point increase on productive transformation investment will generate between 1.23 to 1.65 percentage points of additional economic growth relative to baseline (see Figure A.8).


0.00
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
Response of Debt_GDP
0 1 2 3 4 5 6 7 8 9 10
Lower
DEBT_GDP
Upper


1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
Response of GDP_growth
0 1 2 3 4 5 6 7 8 9 10
Lower
GDP_GROWTH
Upper

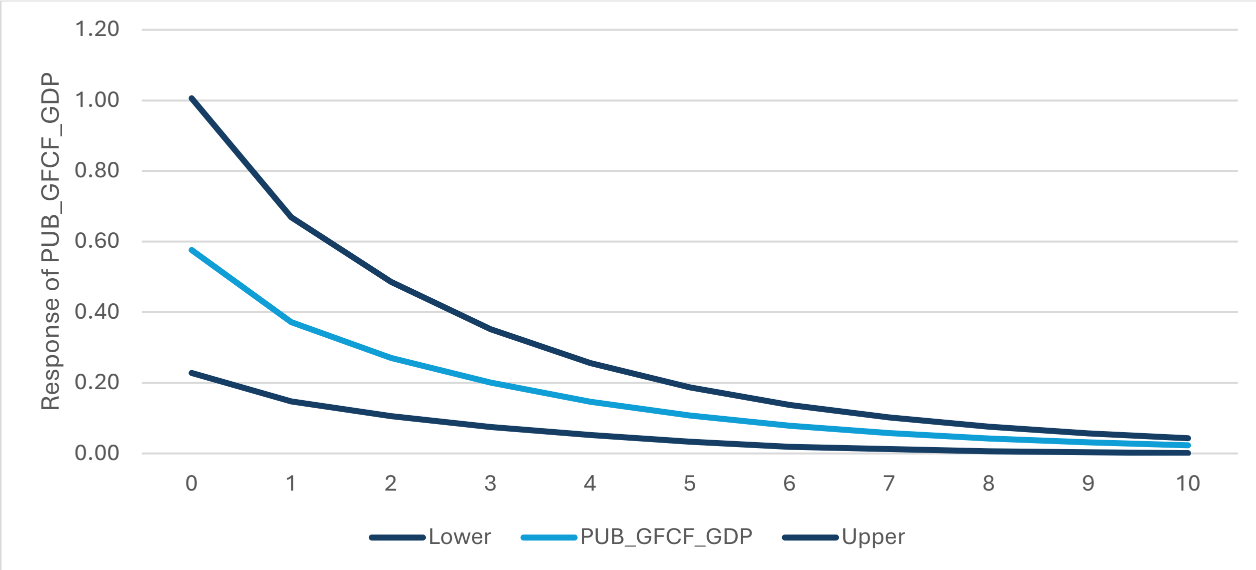


Figure A.8: Impulse Response Functions (IRF) to a Shock in Productive Transformation Investments

Variance Decomposition

Variance decomposition analysis is used within a Vector Autoregression (VAR) framework to determine the proportion of fluctuations in each variable that can be explained by shocks the system over time. While impulse response functions trace the dynamic effect of a specific shock, variance decomposition identifies the relative importance of each shock in explaining movements in the variables included in the model.

The results of the analysis indicate that shocks to productive transformation investments play a significant role in explaining macroeconomic dynamics. In particular, around 10 percent of the forecast error variance of public investments, around 21 percent of debt-to-GDP ratios, and approximately 17 percent real economic growth can be attributed to shocks originating from productive transformation investments (see Table A.3). This suggests that changes in productive transformation investment are an important source of variation in these key economic indicators.

The findings imply that investment directed toward productive transformation has substantial spillover effects across the economy. Its influence extends beyond investment activity itself, affecting fiscal conditions through debt dynamics, while also contributing meaningfully to variations in real economic growth. From a policy perspective, the results highlight the importance of productive transformation investments as a driver of broader macroeconomic performance and debt sustainability.

Forecasting Error Variance explained by the selected public investment expanding innovation (%)

YEAR	PB_GDP	DEBT_GDP	GDP_GROWTH	RIR	XR_GROWTH	PUB_GFCF_GDP
0	5.19	17.63	17.57	0.02	2.48	11.93
5	4.39	21.75	17.16	0.18	2.29	10.18
10	4.17	21.80	16.47	0.22	2.19	9.79
15	4.15	21.61	16.37	0.22	2.18	9.74
20	4.14	21.54	16.36	0.22	2.18	9.74

Table A.3: Variance Decomposition of Productive Transformation Investments Shocks

Results across all methodologies show a clear and consistent message about the importance of productive transformation investments promote economic growth and debt sustainability. All modelling approaches, the time series SUR approach applied to Kenya and the "Synthetic" African Economy as well as the Structural Panel VAR, presents results where the macroeconomic impacts of productive transformation investments lead to higher economic growth and lower debt. This shows the robustness of the result and its relevance as a clear message for African policymakers.

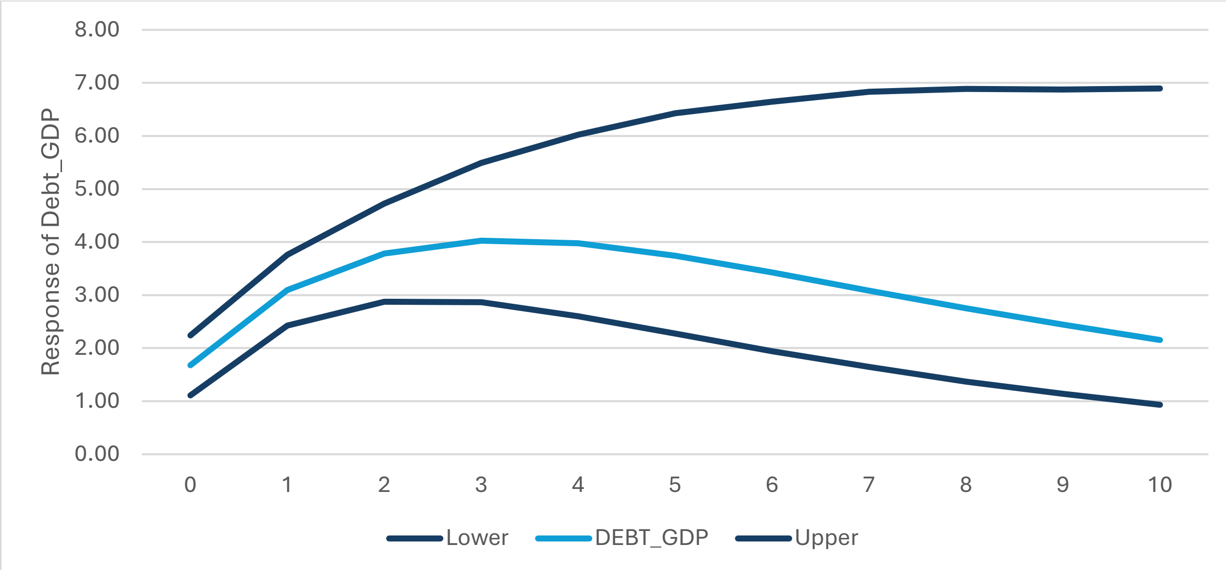
External factors play an important role in driving growth and debt dynamics in Africa. Results from the variance decomposition indicate that Factor 1 explains close to 20 percent of debt-to-GDP ratios, and approximately 5 percent of real economic growth and exchange rate dynamics (see Table A.4). Remember that Factor 1 is a composite index that captures external factors (in particular aggregate global demand) affecting African economies.

Forecasting Error Variance explained by FACTOR 1 innovations (%)

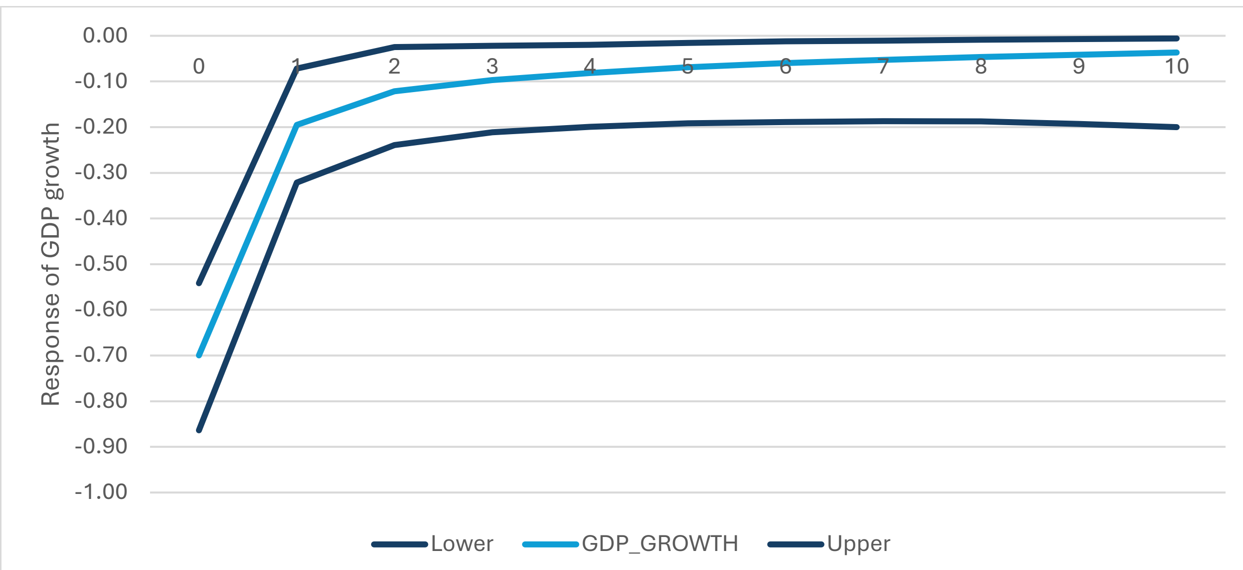
YEAR	FACTOR1	FACTOR2	PB_GDP	DEBT_GDP	GDP_GROWTH	RIR	XR_GROWTH	PUB_GFCF_GDP
0	100.00	0.00	0.10	2.05	4.47	1.45	4.79	0.48
5	100.00	0.00	0.26	12.44	4.89	1.38	4.82	1.24
10	100.00	0.00	0.27	17.97	4.98	1.39	4.83	1.26
15	100.00	0.00	0.27	19.15	4.99	1.39	4.83	1.26
20	100.00	0.00	0.27	19.34	4.99	1.39	4.83	1.26

Table A.4: Variance Decomposition of Productive Transformation Investments Shocks

The impulse response function to shocks in Factor 1 shows that it may reduce debt sustainability by increasing debt to GDP over three percentage points, a lower real GDP growth by 0.7 percentage points, and a higher depreciation of the exchange rate by 5 percentage points. These effects are statistically significant and persistent over time, particularly for debt dynamics where even in year 10 after the shock debt tends to be 2 percentage points higher than in the baseline. Also notice that the risk of the effect on debt to GDP ratio to be higher is significantly greater than the probability of the effect being lower than 2 percentage points higher than the baseline scenario.   However, the effects on GDP growth and exchange rate depreciation tend to be mostly in the short term (see Figure A.9).



Response of Debt_GDP
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0
1
2
3
4
5
6
7
8
9
10
Lower
DEBT_GDP
Upper



Response of GDP growth
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
0
1
2
3
4
5
6
7
8
9
10
Lower
GDP_GROWTH
Upper

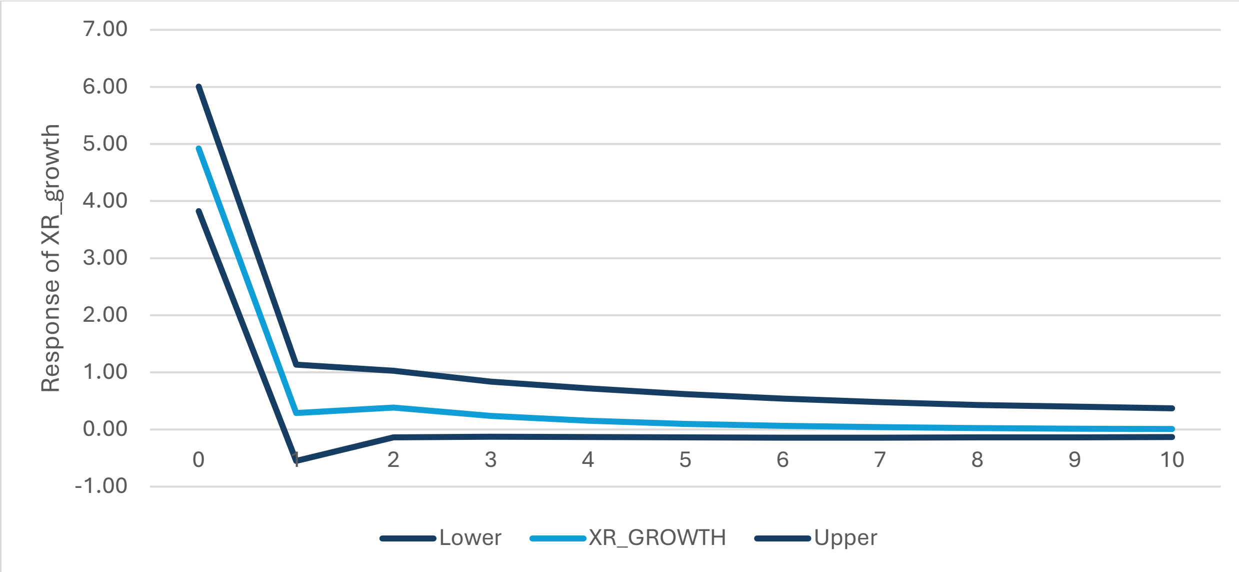


Figure A.9: Impulse Response Functions (IRF) to a Shock in FACTOR 1 (External factors)